Calculating the Deadweight Loss from Taxation in a Small Open Economy

A general method with an application to Sweden

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Introduction

This technical working paper presents a method for estimating the marginal deadweight loss associated with the use of the most important tax policy instruments such as taxes on labour income, indirect taxes on consumption, source-based capital income taxes (mainly the corporation tax), and residence-based capital income taxes (mainly the personal tax on capital income). The paper also proposes a method for quantifying the loss of economic efficiency associated with tax subsidies to certain forms of consumption and the deadweight loss arising from 'non-neutral' capital income taxation.

The calculation of deadweight losses explicitly accounts for the interaction among tax bases, i.e., the fact that a higher tax rate on a certain tax base triggers a behavioural response that tends to reduce not only that tax base itself, but which may reduce other tax bases as well. The model framework underlying the estimations describes a long-run equilibrium in a small open economy where capital is perfectly mobile across borders whereas labour is immobile. With perfect capital mobility, the domestic equilibrium real interest rate is exogenously given from the world capital market, assuming that uncovered interest parity and relative purchasing power parity prevail in the long run. Production requires inputs of capital and labour and takes place under constant returns to scale. Households optimise their labour supply and allocate their consumption over time in accordance with the life cycle theory of consumption. However, it should be stressed that the deadweight loss formulas derived in this paper are more general than the simple life cycle model chosen for purposes of illustration, since the formulas only rely on the general result that the marginal deadweight loss from a tax increase can be measured by the revenue loss caused by the behavioural responses to higher tax rates.

The paper is organised as follows: Sections 1 and 2 describe the household and business sectors, respectively. Section 3 specifies the government budget and derives the effective tax rates on consumption and capita which follow from optimizing household and firm behaviour. In section 4 we derive a set of formulas for the marginal deadweight loss associated with increases in the effective tax rates on labour income, investment, saving and consumption. Section 5 develops a set of formulas for the efficiency loss associated with non-uniform taxation of investment, saving and consumption, and section 6 specifies the data and the elasticity assumptions needed to apply all of the formulas to a particular country. An appendix finally derives formulas for marginal effective tax wedges on business income.

1 The household sector

1.1 Preferences

In accordance with the basic life cycle model of consumer behaviour, the life cycle of the representative consumer is divided into two periods: during the first period she participates in the labour market, and in the second period she is retired and finances consumption by previous savings and by a government transfer. The consumer's lifetime utility is given by the well-behaved utility function

$$U = U\left(C_1, C_2, L\right), \tag{1.1}$$

where L is labour supply during young age, and C_1 and C_2 is total consumption during young and old age, respectively. Total consumption in each period is a CES aggregate of the consumption of housing services (C_H) and the consumption of other goods (C_o), that is:

$$C_i = \left[\mu^{1/\sigma} C_{Hi}^{(\sigma-1)/\sigma} + \left(1-\mu\right)^{1/\sigma} C_{oi}^{(\sigma-1)/\sigma}\right]^{\frac{\sigma}{\sigma-1}}, \quad 0 < \mu < 1, \quad i = 1, 2, \tag{1.2}$$

where σ is the elasticity of substitution between housing and other goods. The consumption of housing is in turn a CES aggregate of services from owner-occupied housing (*H*) and services from rental housing (*h*), with a substitution elasticity σ_h between the two types of housing:

$$C_{Hi} = \left[\eta^{1/\sigma_h} H_i^{(\sigma_h-1)/\sigma_h} + \left(1-\eta\right)^{1/\sigma_h} h_i^{(\sigma_h-1)/\sigma_h}\right]^{\frac{\sigma_h}{\sigma_h-1}}, \quad 0 < \eta < 1, \quad i = 1, 2. \tag{1.3}$$

Similarly, the consumption of other goods is an aggregate of *N* different consumer goods and services with a common substitution elasticity σ_o among them:

$$C_{oi} = \left[\sum_{n=1}^{N} \beta_n^{1/\sigma_o} x_{ni}^{(\sigma_o - 1)/\sigma_o} \right]^{\frac{\sigma_o}{\sigma_o - 1}}, \qquad \sum_{n=1}^{N} \beta_n = 1, \qquad i = 1, 2. \quad (1.4)$$

In so far as the individual goods x_n in the consumption aggregate (1.4) are tradeable internationally, we adopt the usual small-open-economy assumption that foreign and domestically-produced variants of these good are perfect substitutes. Hence there is no need to distinguish imported from domestically-produced goods in (1.4).

1.2 The optimal composition of consumption

A necessary condition for utility maximization is that the expenditure needed to generate any given amount of utility be minimized. Thus the optimal allocation of consumption between housing and other goods in each period is found as the solution to the problem:

$$\underset{C_H,\, C_o}{\text{Minimize}} \quad PC \equiv P_H C_H + P_o C_o \quad \text{subject to (1.2)}, \qquad (1.5)$$

where P is an aggregate consumer price index and P_H and P_O are the price indices for housing and other goods, respectively, and where we have dropped the time subscripts for convenience. The solution to this problem implies that

$$C_H = \mu \left(\frac{P_H}{P} \right)^{-\sigma} C, \qquad (1.6)$$

$$C_o = (1 - \mu) \left(\frac{P_o}{P} \right)^{-\sigma} C, \qquad (1.7)$$

$$P = \left[\mu P_H^{1-\sigma} + (1 - \mu) P_o^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \qquad (1.8)$$

where C is given by (1.2). In a similar way, the solution to the expenditure minimization problems

$$\underset{H,\, h}{\text{Minimize}} \quad P_H C_H \equiv p_H H + p_h h \quad \text{subject to (1.3)}, \qquad (1.9)$$

$$\underset{x_n}{\text{Minimize}} \quad P_o C_o \equiv \sum_{n=1}^{N} p_n x_n \quad \text{subject to (1.4)}, \tag{1.10}$$

can be shown to imply that

$$H = \eta \left(\frac{p_H}{P_H} \right)^{-\sigma_h} C_H, \tag{1.11}$$

$$h = (1-\eta) \left(\frac{p_h}{P_H} \right)^{-\sigma_h} C_H, \tag{1.12}$$

$$P_H = \left[\eta p_H^{1-\sigma_h} + (1-\eta) p_h^{1-\sigma_h} \right]^{\frac{1}{1-\sigma_h}}, \tag{1.13}$$

$$x_n = \beta_n \left(\frac{p_n}{P_o} \right)^{-\sigma_o} C_o, \qquad n = 1, \ldots, N, \tag{1.14}$$

$$P_o = \left[\sum_{n=1}^{N} \beta_n p_n^{1-\sigma_o} \right]^{\frac{1}{1-\sigma_o}}, \tag{1.15}$$

where p_H and p_h are the user costs of owner-occupied and rental housing services, respectively, and p_n is the consumer price of the non-housing good n.

1.3 The household budget constraint

Perfect substitutability between domestic and foreign traded goods means that the producer prices of these goods are exogenously given from the world market. We normalize all of these producer prices to one. The assumptions on production technologies etc. stated in Section 2 imply that the long-run marginal costs of producing domestic non-traded goods are independent of the scale of output. We may therefore choose our units of measurement such that the producer prices of the domestic non-traded goods entering the consumption aggregate (1.4) are likewise equal to one. Indirect taxes are levied at identical rates on domestic and imported goods in accordance with the destination principle. The consumer price of the non-housing good n is therefore given as

$$p_n = 1 + t^n, \qquad n = 1,....,N, \tag{1.16}$$

where t^n is the indirect tax rate on good *n*, measured on a tax-exclusive ad-valorem basis.

We assume that a non-housing good may be transformed into a unit of housing at a constant marginal rate of transformation (MRT) equal to one. The role of the domestic 'construction industry' is to organise this process by buying non-housing goods, transforming them into housing units, and selling or renting these dwellings to consumers. With MRT=1, a long-run housing market equilibrium involving zero profits for 'construction' firms requires that the producer price of a newly 'constructed' (transformed) unit of housing is equal to one. If the sale of new housing units is subject to the indirect tax rate t^H (which could be a VAT), the cost to consumers of purchasing a new unit of housing will then be $1+t^H$ and in a long-run housing market equilibrium this is also the price of an existing unit of housing, since buying an existing dwelling or having a 'construction firm' deliver a new one must be equally attractive for consumers. If *r* is the pre-tax real interest rate, t^r is the effective capital income tax rate levied on the real return to financial saving, τ^H is the effective property tax rate on owner-occupied housing, and δ is the rate of housing depreciation, the user cost of a unit of owner-occupied housing may then be written as

$$p_H = \left(1+t^H\right)\left[r\left(1-t^r\right)+\tau^H+\delta\right]. \tag{1.17}$$

For a consumer financing the purchase of a house by equity, the amount $\left(1+t^H\right)r\left(1-t^r\right)$ is the after-tax opportunity cost of investing her wealth in housing rather than in financial assets. In the case of debt finance, this same amount represents the after-tax cost of debt service, assuming that the tax code allows deductibility for mortgage interest payments. The amount $\left(1+t^H\right)\delta$ in (1.17) is the expenditure needed to maintain the original value of a unit of housing over the period considered. Finally, the user cost of owner-occupied housing includes the property tax bill $\left(1+t^H\right)\tau^H$, where τ^H is the effective property tax rate, defined as the actual property tax paid relative to the current market value of the house.

The consumer price of a unit of rental housing service is

$$p_h = \left(1+t^H\right)\left(r+\tau^h+\delta\right). \tag{1.18}$$

The amount $\left(1+t^H\right)\left(r+\delta\right)$ reflects the landlord's cost of finance and maintenance, while the term $\left(1+t^H\right)\tau^h$ captures any specific indirect tax on rental housing. If rental housing is subsidized in one way or another, the effective tax rate τ^h is negative.

When analysing the marginal deadweight loss from the taxation of labour income, we will consider the effect of a proportional increase in the marginal tax rates of all taxpayers. For this purpose we may approximate the tax-transfer schedule faced by the working population by the linear tax schedule

$$T = t^w WL - B_1, \qquad 0 < t^w < 1, \qquad B_1 > 0, \tag{1.19}$$

where *W* is the real producer wage rate (the employer's real labour cost), t^w is the effective marginal tax rate on labour income (including social security taxes as well as personal income tax), and B_1 is a lump-sum transfer to people of working age. Note that although the marginal tax rate is constant, (1.19) implies that taxation is progressive in the sense that the average tax rate $T/WL = t^w - B_1/WL$ is increasing in total labour income *WL*. In the empirical application of the model t^w should be estimated as a weighted average of the effective marginal labour income tax rates across all taxpayers.

Since we do not consider taxes on bequests (which generate very little revenue in most countries), we assume that the representative young household starts out in period 1 without zero initial wealth. At the start of the period the household borrows to acquire a stock of owner-occupied housing H_1 , but it also consumes an amount of rental housing services h_1 during young age. Denoting net financial saving during period 1 by S^f and using (1.19), we may thus specify the household budget constraint for that period as

$$S^f + \left(1+t^H\right)H_2 = \left(1-t^w\right)WL + B_1 - \left(1+t^H\right)\left[r\left(1-t^r\right)+\tau^H+\delta\right]H_1 - p_h h_1 - \sum_{n=1}^{N} p_n x_{n1}, \tag{1.20}$$

where H_2 is the stock of owner-occupied housing carried over the second period of life. During that period, the household finances its expenses by the return to its financial saving, by a government transfer to retirees (B_2), and by realizing its financial and housing assets at the end of the period. Hence the budget constraint for period 2 is[1]

$$\left(1+t^H\right)\left(\tau^H+\delta\right)H_2+p_h h_2+\sum_{n=1}^{N}p_n x_{n2}=\left[1+r\left(1-t^r\right)\right]S^f+\left(1+t^H\right)H_2+B_2. \quad (1.21)$$

Using the definition (1.17) of the user cost of owner-occupied housing, we may rewrite (1.21) as:

$$p_H H_2+p_h h_2+\sum_{n=1}^{N}p_n x_{n2}=\left[1+r\left(1-t^r\right)\right]\left[S^f+\left(1+t^H\right)H_2\right]+B_2. \quad (1.22)$$

Combining (1.20) and (1.22), we obtain the consumer's intertemporal budget constraint:

$$\begin{aligned} &p_H H_1+p_h h_1+\sum_{n=1}^{N}p_n x_{n1}+p\left(p_H H_2+p_h h_2+\sum_{n=1}^{N}p_n x_{n2}\right) \\ &=\left(1-t^w\right)WL+B_1+pB_2, \qquad p\equiv\frac{1}{1+r\left(1-t^r\right)}, \end{aligned} \quad (1.23)$$

where p is the relative price of future consumption in terms of present consumption. Exploiting the definitions of aggregate consumer spending given in (1.5), (1.9) and (1.10), we may finally condense the budget constraint (1.23) in the following way:

$$P\left(C_1+pC_2\right)=\left(1-t^w\right)WL+B_1+pB_2 \quad \Leftrightarrow$$

$$C_1+pC_2=wL+I, \qquad w\equiv\frac{\left(1-t^w\right)W}{P}, \qquad I\equiv\frac{B_1+pB_2}{P}, \quad (1.24)$$

where w is the after-tax real consumer wage, and I is the (present value of the) household's exogenous real income.

[1] Note that we are assuming the interest rate as well as all tax rates (and hence all consumer prices) to be constant over time, so these variables carry no time subscripts.

1.4 Optimal labour supply and savings

To maximize lifetime utility the household must allocate consumption across the different goods and services in accordance with (1.6), (1.7), (1.11), (1.12) and (1.14). In addition, the household must choose a labour supply and an intertemporal allocation of aggregate consumption that maximizes lifetime utility (1.1) subject to the lifetime budget constraint (1.24). The solution to the latter problem yields a labour supply function and consumption functions of the following form:

$$L = L(w, p, I), \qquad C_1 = C_1(w, p, I), \qquad C_2 = C_2(w, p, I). \quad (1.25)$$

Defining real aggregate savings as

$$S \equiv \frac{S^f + (1+t^H)H_2}{P}, \quad (1.26)$$

we can also use (1.5), (1.9), (1.10), (1.22), (1.25) and (1.26) to derive the savings function:

$$S = p\left[C_2(w, p, I) - \frac{B_2}{P}\right]. \quad (1.27)$$

The allocation of total savings between financial saving and housing investment follows from (1.6), (1.11) and (1.27).

In most countries a large fraction of financial saving is channelled through institutional investors such as pension funds and life insurance companies. The returns to saving through these intermediaries are typically subject to favourable tax rules. At the same time this form of saving is less liquid than 'free' savings via bank accounts and via direct purchases of bonds and stocks etc. The two forms of saving are therefore likely to be imperfect substitutes in the eyes of consumers. To capture this, we specify total financial saving as the following CES aggregate of 'institutional' saving (S^I) and 'free' saving (S^F):

$$S^f = \left[\gamma^{-1/\phi}\left(S^I\right)^{(\phi+1)/\phi} + (1-\gamma)^{-1/\phi}\left(S^F\right)^{(\phi+1)/\phi}\right]^{\frac{\phi}{\phi+1}}, \qquad 0 < \gamma < 1. \quad (1.28)$$

Recalling that r is the pre-tax return to financial saving and that t^r is an 'aggregate' effective tax rate on its return, the household's total after-tax income from financial saving is given by

$$r\left(1-t^r\right)S^f = r\left(1-t^I\right)S^I + r\left(1-t^F\right)S^F, \qquad (1.29)$$

where t^I and t^F are the effective tax rates on the real returns to institutional and free savings, respectively. To maximize utility, the household must allocate any given total amount of financial saving across the two components S^I and S^F so as to maximize the total after-tax financial income specified in (1.29), subject to the constraint given by (1.28). The solution to this problem yields:

$$S^I = \left(\frac{1-t^I}{1-t^r}\right)^{\phi} \gamma S^f, \qquad (1.30)$$

$$S^F = \left(\frac{1-t^F}{1-t^r}\right)^{\phi} \left(1-\gamma\right) S^f, \qquad (1.31)$$

$$1-t^r = \left\{\gamma\left(1-t^I\right)^{\phi+1} + \left(1-\gamma\right)\left(1-t^F\right)^{\phi+1}\right\}^{\frac{1}{\phi+1}}. \qquad (1.32)$$

Equation (1.32) determines the aggregate effective capital income tax rate t^r as a function of the effective tax rates on the two forms of financial saving, given that households optimize their portfolio composition.

2 The business sector

2.1 Technology

The domestic business sector includes 'production firms' and 'trading firms'. The output of the competitive domestic production firms is assumed to be a perfect substitute for foreign-produced goods which sell in the world market at a constant producer price equal to one.

Trading firms buy the output of production firms and transform it into the different variants of consumer goods entering the consumption aggregate (1.4). The marginal rate of transformation is constant and equal to one. There are no economies of scale in the trading sector, so free entry ensures that trading firms end up with zero profits in long-run equilibrium. Since the only inputs used by trading firms are the goods purchased from production firms which are 'transformed' and sold to consumers with a zero profit margin, the trading sector generates no net income in long-run equilibrium and may thus be ignored in all of the following analysis.[2]

Domestic output (Y) is produced by combining an aggregate capital good K with domestic labour L in the well-behaved production function:

$$Y = F(K,L), \qquad F_K > 0, \quad F_L > 0,$$
$$F_{KK} < 0, \quad F_{LL} < 0, \quad F_{KL} = F_{LK} > 0. \tag{2.1}$$

[2] Our simple story about the trading firms just serves to explain how the production and distribution of goods may take place under perfectly competitive conditions even though consumers end up consuming goods of different varieties. In this way we avoid introducing complications arising from imperfect competition.

The function $F(K,L)$ is homogeneous of degree one (constant returns to scale), and the subscripts in (2.1) indicate partial derivatives.

2.2 Factor demands

If *c* is the cost of capital net of depreciation and δ_k is the rate of depreciation of business capital, the net profit Π of the representative domestic production firm is

$$\Pi = Y - (c + \delta_k) K - WL, \tag{2.2}$$

where we recall that *W* is the employer's real labour cost measured in producer prices. The representative competitive production firm maximizes (2.2) subject to (2.1), yielding the standard first-order conditions that the marginal product of capital should equal the total user cost of capital *gross* of depreciation (ρ) and that the marginal product of labour should equal the real product wage:

$$F_K(K,L) = \rho, \qquad \rho \equiv c + \delta_k, \tag{2.3}$$

$$F_L(K,L) = W. \tag{2.4}$$

From the labour supply function stated in (1.25) we have:

$$L = L\left(\frac{W(1-t^w)}{P}, \frac{1}{1+r(1-t^r)}, I\right). \tag{2.5}$$

As we shall see in section 2.4, the cost of capital (*c*) is determined by the exogenous world interest rate (*r*) and by the parameters of the tax system. The consumer price index (*P*) is determined by the exogenous producer prices and the various indirect tax rates, as laid out in detail in section 3, and the present value of transfers (*I*) and the marginal income tax rates t^w and t^r are likewise exogenous. Given these parameters, the system (2.3) through (2.5) then determines *K*, *L*, and *W*. To calculate marginal deadweight losses, we will need to know how the direct and indirect tax rates (with the latter working through *P*) affect these variables. Exploiting the homogeneity properties of the production function, one can derive the following results:

Effects of a change in t^w :

$$\frac{dL}{L} = \frac{dK}{K} = -\varepsilon_w^L \cdot \left(\frac{dt^w}{1-t^w} \right), \qquad \varepsilon_w^L \equiv \frac{\partial L}{\partial w} \frac{w}{L}. \quad (2.6)$$

Effects of a change in P:

$$\frac{dL}{L} = \frac{dK}{K} = -\varepsilon_w^L \cdot \left(\frac{dP}{P} \right). \quad (2.7)$$

Effects of a change in t^r :

$$\frac{dL}{L} = \frac{dK}{K} = -\varepsilon_r^L \cdot \left(\frac{dt^r}{1-t^r} \right), \qquad \varepsilon_r^L \equiv \frac{\partial L}{\partial r\left(1-t^r\right)} \frac{r\left(1-t^r\right)}{L}. \quad (2.8)$$

Effects of a change in ρ :

$$\frac{dL}{L} = -\varphi \varepsilon_w^L \cdot \left(\frac{d\rho}{\rho} \right), \qquad \varphi \equiv \frac{\rho K}{WL}, \quad (2.9)$$

$$\frac{dK}{K} = -\left(\varepsilon_\rho^K + \varphi \varepsilon_w^L\right)\left(\frac{d\rho}{\rho} \right), \qquad \varepsilon_\rho^K \equiv -\frac{\partial K}{\partial \rho} \frac{\rho}{K} = -\frac{1}{F_{KK}} \frac{\rho}{K}. \quad (2.10)$$

The first equalities in (2.6) through (2.8) follow from the fact that, under constant returns to scale, the marginal products of capital and labour depend only on the capital-labour ratio. According to (2.3) K/L must therefore be constant – implying $dL/L=dK/K$ - as long as the user cost of capital is constant. In other words, when labour supply changes as a result of a change in t^w, P or t_r, the capital stock must change by the same relative amount, and these changes in factor inputs will depend on the elasticities of labour supply with respect to the real after-tax wage rate and the real after-tax interest rate, as stated in (2.6) through (2.8).

To understand the results in (2.9) and (2.10), note that on impact, at the given initial level of labour supply L, it follows from (2.3) that a unit increase in the cost of capital changes the optimal capital stock by the amount

$$\frac{\partial K}{\partial \rho} = \frac{1}{F_{KK}}. \tag{2.11}$$

If the change in the user cost of capital is $d\rho$, eqs. (2.4) and (2.11) imply that the initial impact on the producer real wage is

$$dW = F_{LK} \cdot \frac{\partial K}{\partial \rho} \cdot d\rho = \frac{F_{KL}}{F_{KK}} \cdot d\rho, \tag{2.12}$$

where we have used the fact that $F_{LK} = F_{KL}$. Since the production function $F(K, L)$ is homogeneous of degree one, the marginal product function $F_K(K, L)$ is homogeneous of degree zero, implying

$$K \cdot F_{KK} + L \cdot F_{KL} = 0 \quad \Leftrightarrow \quad \frac{F_{KL}}{F_{KK}} = -\frac{K}{L}, \tag{2.13}$$

so that (2.12) becomes

$$dW = -\frac{K}{L} \cdot d\rho. \tag{2.14}$$

As stated in (2.5), the marginal real consumer wage governing labour supply is $w \equiv W\left(1 - t^w\right)/P$, so from (2.14) we get

$$dw = \left(\frac{1 - t^w}{P}\right) dW = -\left(\frac{1 - t^w}{P}\right) \frac{K}{L} \cdot d\rho. \tag{2.15}$$

Hence the change in labour supply induced by the change in the user cost of capital is

$$dL = \frac{\partial L}{\partial w} \cdot dw = -\left(\frac{1-t^w}{P}\right)\frac{K}{L}\frac{\partial L}{\partial w} \cdot d\rho, \qquad (2.16)$$

which in turn leads to (2.9). Ceteris paribus, this drop in labour supply induces a similar relative drop in capital input as firms adjust the capital stock to maintain the optimal capital-labour ratio, so from (2.9) we get the

Indirect effect on K of a change in ρ:

$$\left(\frac{dK}{K}\right)_{\text{indirect}} = -\varphi\varepsilon_w^L \cdot \left(\frac{d\rho}{\rho}\right), \qquad \varphi \equiv \frac{\rho K}{WL}. \qquad (2.17)$$

In addition to this indirect effect of the rise in ρ on K, there is the direct immediate effect given by (2.11) which may be written as follows:

Direct effect on K of a change in ρ:

$$\left(\frac{dK}{K}\right)_{\text{direct}} = -\varepsilon_\rho^K \cdot \frac{d\rho}{\rho}, \qquad \varepsilon_\rho^K \equiv -\frac{\partial K}{\partial \rho}\frac{\rho}{K} = -\frac{1}{F_{KK}}\frac{\rho}{K} > 0, \qquad (2.18)$$

where ε_ρ^K is the (numerical) elasticity of capital demand with respect to the user cost, calculated at the given initial level of labour supply. Adding the direct and the indirect effects in (2.18) and (2.17), we end up with (2.10). The important insight from this analysis is that a (tax-induced) rise in the cost of capital reduces labour supply because the fall in capital demand reduces the pre-tax wage rate, and the drop in labour supply in turn amplifies the initial drop in investment.

2.3 The choice of organizational form

To highlight how the tax system may distort the choice of organizational form, we may think of the aggregate business capital stock K as consisting of capital owned by different forms of business organization. In a small open economy, an important distinction is that between widely and closely held firms. In the present context, widely held firms are defined as firms which can obtain equity and debt finance in the international capital market, whereas closely held firms must finance investment from domestic sources. For widely held firms, the cost of finance is thus given by r which we have so far referred to as the real world interest rate. However, in the case of equity finance r may be thought of as the risk-adjusted real return to equity required by international stock markets. A crucial point is that the cost of finance for widely held firms is unaffected by domestic personal taxes on interest, dividends and capital gains, since these taxes are residence-based and hence do not apply to the international investors whose behaviour determines r. By contrast, for closely held firms relying on funding from domestic residents, the cost of finance may be affected by domestic personal taxes, as we shall explain in section 2.4.

Widely held firms will typically be large corporations with many shareholders, while closely held firms will typically be small corporations and proprietorships/partnerships with a few or a single owner. Since the legal and institutional framework and/or the governance problems associated with the two types of organizational form are different (see, e.g., the discussion by Hagen and Sørensen (1998)), we may think of capital invested in the different types of business organization as being imperfect substitutes. Thus we specify the total stock of business capital as a CES-aggregate of capital invested in widely held firms (K_w) and capital invested in closely held firms (K_c), with a substitution elasticity σ_k between them:

$$K = \left[\varpi^{1/\sigma_k} K_w^{(\sigma_k - 1)/\sigma_k} + \left(1 - \varpi\right)^{1/\sigma_k} K_c^{(\sigma_k - 1)/\sigma_k} \right]^{\frac{\sigma_k}{\sigma_k - 1}}, \qquad 0 < \varpi < 1. \quad (2.19)$$

In a similar way, we assume that the capital stock invested in closely held firms is a CES-aggregate of capital invested in closely held corporations (K_{cc}) and capital invested in proprietorships/partnerships (K_{cp}):

$$K_c = \left[\chi^{1/\sigma_c} K_{cc}^{(\sigma_c - 1)/\sigma_c} + \left(1 - \chi\right)^{1/\sigma_c} K_{cp}^{(\sigma_c - 1)/\sigma_c} \right]^{\frac{\sigma_c}{\sigma_c - 1}}, \qquad 0 < \chi < 1. \quad (2.20)$$

Following the same method, one could disaggregate the capital stocks in each subsector into different asset types to allow for the possibility that different assets may be subject to different tax rules. However, in this paper we choose to focus only on the implications of asymmetric tax treatment of different organizational forms.

In parallel to the specification of the consumer price indices, the user costs associated with the aggregates K, K_c, and K_w (denoted by ρ, ρ_c, and ρ_w, respectively) are defined so as to satisfy

$$\rho K = \rho_w K_w + \rho_c K_c, \tag{2.21}$$

$$\rho_c K_c = \rho_{cc} K_{cc} + \rho_{cp} K_{cp}, \tag{2.22}$$

where ρ_{cc} is the user cost of capital in closely held corporations, and ρ_{cp} is the user cost in closely held unincorporated firms.

The disaggregation outlined above involves treating the representative firm as a conglomerate that spreads out its total capital stock across different organizational forms. To maximize profits, the conglomerate must minimize the total user cost ρK associated with the use of any given aggregate stock of capital, that is, it must allocate its capital across widely and closely held firms so as to minimize (2.21) subject to (2.19), where K is treated as fixed. Solving this problem yields:

$$K_w = \varpi \left(\frac{\rho_w}{\rho} \right)^{-\sigma_k} K, \tag{2.23}$$

$$K_c = (1-\varpi) \left(\frac{\rho_c}{\rho} \right)^{-\sigma_k} K, \tag{2.24}$$

$$\rho = \left[\varpi \rho_w^{1-\sigma_k} + (1-\varpi) \rho_c^{1-\sigma_k} \right]^{\frac{1}{1-\sigma_k}}. \tag{2.25}$$

In a similar vein, cost minimization with respect to the choice of the alternative organizational forms within the sector for closely held firms implies:

$$K_{cc} = \chi \left(\frac{\rho_{cc}}{\rho_c} \right)^{-\sigma_c} K_c, \tag{2.26}$$

$$K_{cp} = (1-\chi) \left(\frac{\rho_{cp}}{\rho_c} \right)^{-\sigma_c} K_c, \tag{2.27}$$

$$\rho_c = \left[\chi \rho_{cc}^{1-\sigma_c} + (1-\chi) \rho_{cp}^{1-\sigma_c} \right]^{\frac{1}{1-\sigma_c}}. \tag{2.28}$$

2.4 The cost of capital and the investment and savings tax wedges

To estimate how the tax system affects the user cost of capital, one has to allow for the impact of the statutory tax rate as well as the rules defining the business income tax base. Building on the approach of King and Fullerton (1984), the appendix derives formulas for the cost of capital for widely and closely held firms. For widely held corporations which can fund themselves in the international capital market, the appendix shows that the cost of capital associated with an *equity-financed* investment is given by

$$\rho_w^e - \delta_k = \left(\frac{r}{1-\tau}\right)(1-\tau a), \tag{2.29}$$

where τ is the statutory corporate income tax rate, and a is the present value of capital allowances generated by an additional unit of investment in *excess* of allowances for true economic depreciation. Formula (2.29) thus accounts for the fact that tax systems typically allow accelerated depreciation and sometimes also offer additional investment incentives such as investment tax credits etc. The appendix explains how one may estimate the size of a, given the actual depreciation rules and an estimate of the true rate of economic depreciation of the asset considered. When depreciation for tax purposes corresponds to true economic depreciation, we see from (2.29) that the cost of capital is simply $r/(1-\tau)$, since such a pre-tax rate of return will ensure that the true economic after-tax profit per unit of investment is just equal to the real cost of finance, given by the international real interest rate r (here we abstract from the risk premium normally required on equity, thus measuring rates of return in risk-adjusted terms). In the presence of accelerated depreciation ($a > 0$), the depreciation rules involve a tax subsidy to investment which is seen from (2.29) to lower the cost of capital.

In the case of *debt finance* where interest payments are deductible from the corporate income tax base, the appendix shows that the cost of capital for widely held firms becomes

$$\rho_w^d - \delta_k = r - \left(\frac{\tau}{1-\tau}\right)(\pi + ra), \tag{2.30}$$

where π is the rate of inflation. Conventional tax systems involve a tax subsidy to debt finance by allowing deductibility of the full nominal interest expense, including the inflation premium that just serves to offset the erosion of the real debt burden caused by inflation. Accordingly, we see from (2.30) that a higher inflation rate will ceteris paribus lower the cost of capital.

As mentioned, the relevant discount rate (the cost of finance) for widely held corporations is simply the world interest rate which is assumed to determine the risk-adjusted rate of return on shares required by international investors. For closely held corporations the situation is different since these firms are assumed to obtain their equity finance from domestic investors subject to domestic personal taxes. Such an investor will be willing to invest equity in a domestic closely held firm as long as the risk-adjusted real return to equity before tax (r^e) implies an after-tax return which is at least as high as the real after-tax return obtainable on financial saving, that is, as long as

$$r^e\left(1-t^e\right)=r\left(1-t^r\right), \tag{2.31}$$

where t^e is the effective marginal personal tax rate on the return to shares, and $r\left(1-t^r\right)$ is given by (1.32). Because dividends and capital gains on shares may be subject to special tax rules, we may have $t^e \neq t^r$, so according to (2.31) the required pre-tax return on equity in closely held firms may deviate from the pre-tax interest rate *r*. Thus changes in the effective tax rate t^r on financial saving (e.g. a change in the personal tax rate on interest income) and changes in personal tax rates on dividends and capital gains will affect the cost of finance for closely held firms, and via this channel personal taxes will influence the user cost of capital for such firms. Using Sweden as an example, the appendix shows how one may derive the user costs for proprietorships and closely held corporations, accounting for the special tax rules for these firms while applying the same general approach that led to the user cost for widely held firms in (2.29) and (2.30).

The difference between the real pre-tax return to investment $(\rho-\delta_k)$ and the real after-tax return to saving $\left(r\left(1-t^r\right)\right)$ is usually referred to as the marginal effective tax wedge. Since the real pre-tax interest rate *r* is exogenously determined from the world capital market, it is useful to split the total tax wedge into an 'investment tax wedge' and a 'savings tax wedge'. The savings tax wedge is the difference between the pre-tax and the after-tax return to saving, that is,

$$\text{Savings tax wedge:} \quad r - r\left(1 - t^r\right) = t^r r. \tag{2.32}$$

Given the exogenous world real interest rate, we see that the savings tax wedge is determined solely by the effective capital income tax rate t^r. The investment tax wedge is the difference between the pre-tax return to investment $(\rho - \delta_k)$ and the international cost of finance (r). For our three organizational forms (denoted by the subscripts introduced in section 2.3), we thus have the

Investment tax wedges:

$$t_w^k \equiv \rho_w - \delta_k - r, \tag{2.33}$$

$$t_{cc}^k \equiv \rho_{cc} - \delta_k - r, \tag{2.34}$$

$$t_{cp}^k \equiv \rho_{cp} - \delta_k - r. \tag{2.35}$$

Note that even though it reduces the demand for capital, any increase in a source-based tax on 'active' business investment in the domestic economy will translate fully into a similar increase in the domestic user cost of capital (ρ), since the constancy of the real interest rate means that the fall in capital demand does not reduce the cost of finance. From (2.29), (2.30) and (2.33) it follows that the investment tax wedges for widely held firms for the two modes of finance (indicated by the superscripts e for equity and d for debt) are

$$t_w^{ke} = \frac{\tau r\left(1-a\right)}{1-\tau}, \qquad t_w^{kd} = -\frac{\tau\left(\pi + ra\right)}{1-\tau}. \tag{2.36}$$

We see from (2.36) that the domestic residence-based capital income tax rate t^r has no impact on the investment tax wedge and hence no impact on investment incentives for widely held firms. This illustrates the importance of distinguishing between taxes on saving and taxes on investment in a small open economy.

3 The government sector

3.1 Net public revenue

To estimate the deadweight loss from taxation, we need to calculate the present value of the taxes paid by each cohort of taxpayers over the life cycle. In doing so, we invoke the assumption that the economy is in a long-run equilibrium where all capital stocks (including the housing stock) are constant. The construction of new housing units in period *i* is then equal to the depreciation of the existing housing stock, $\delta(H_i + h_i)$, so the revenue from the indirect tax on the sale of new housing units is $t^H \delta(H_1 + h_1)$. As shown in section 2.4, the source-based business taxes included in the investment tax wedges defined in (2.37) through (2.39) will translate into an equivalent increase in the user cost of capital. Via the resulting fall in domestic investment, the burden of these taxes will therefore be fully shifted to domestic workers through a fall in domestic wage rates, as implied by (2.15). Since taxpayers are active in the labour market during the first period of their life cycle, we assign the revenue from the investment taxes to the first period when calculating the present value of tax revenue, that is, we do not discount the revenue from these taxes. Given the various taxes and transfers introduced in sections 1 and 2, the present value of the total net payments from each cohort of taxpayers to the government then becomes equal to the following expression, where the term $-(1+t^H)t^r r$ reflects the revenue loss from the mortgage interest deductions taken by young households:

$$R=\sum_{n=1}^{N}t^{n}x_{n1}+t^{H}\delta\left(H_{1}+h_{1}\right)+\left(1+t^{H}\right)\left[\left(\tau^{H}-t^{r}r\right)H_{1}+\tau^{h}h_{1}\right]$$

$$+t^{w}WL-B_{1}+t_{w}^{k}K_{w}+t_{cc}^{k}K_{cc}+t_{cp}^{k}K_{cp}$$

$$+\frac{1}{1+r\left(1-t^{r}\right)}\left[\sum_{n=1}^{N}t^{n}x_{n2}+t^{H}\delta\left(H_{2}+h_{2}\right)+\left(1+t^{H}\right)\left(\tau^{H}H_{2}+\tau^{h}h_{2}\right)+t^{r}rS^{f}-B_{2}\right]. \quad (3.1)$$

3.2 Effective indirect tax rates

It will be convenient to condense the above expression for net public revenue. For this purpose, we introduce the effective ad-valorem tax-inclusive indirect tax rate on 'other goods', t_o^c, defined to satisfy the constraint

$$t_{o}^{c}P_{o}C_{oi}=\sum_{n=1}^{N}t^{n}x_{ni},\qquad i=1,2. \quad (3.2)$$

Inserting (1.14) through (1.16) into (3.2) and dropping the time subscript i for convenience, we get

$$t_{o}^{c}P_{o}=\sum_{n=1}^{N}t^{n}\beta_{n}p_{n}^{-\sigma_{o}}P_{o}^{\sigma_{o}}\quad\Leftrightarrow\quad t_{o}^{c}=\sum_{n=1}^{N}t^{n}\beta_{n}p_{n}^{-\sigma_{o}}P_{o}^{-(1-\sigma_{o})}. \quad (3.3)$$

Using (1.15) plus the fact that $p_n=1+t^n$, we may rewrite the last equality in (3.3) as

$$t_{o}^{c}=\sum_{n=1}^{N}\tilde{\beta}_{n}\cdot\frac{t^{n}}{1+t^{n}},\qquad\tilde{\beta}_{n}\equiv\frac{\beta_{n}\left(1+t^{n}\right)^{1-\sigma_{o}}}{\sum_{n}\beta_{n}\left(1+t^{n}\right)^{1-\sigma_{o}}}. \quad (3.4)$$

Equation (3.4) defines the effective indirect tax rate on 'other goods', given that consumers minimize the expenditure needed to obtain a given level of utility from the consumption of these goods. Since $\sum_{n}\tilde{\beta}_{n}=1$ and $p_n=1+t^n$, we see that t_o^c is a weighted average of the ad valorem tax rates t^n/p_n on the individual goods in the category of 'other goods'. Note that if all goods are taxed at the same uniform tax exclusive rate , (3.4) implies that $t^n=t$

which is just the standard formula for the translation of a tax-exclusive into a tax-inclusive indirect tax rate.

In a similar way, we may define an 'aggregate' effective indirect tax rate on housing consumption (t_h^c) satisfying

$$t_H^c P_H C_{Hi} = t^H \delta\left(H_i + h_i\right) + \left(1+t^H\right)\left[\left(\tau^H - t^r r\right) H_i + \tau^h h_i\right], \qquad i = 1,2. \tag{3.5}$$

Inserting (1.6) plus (1.11) through (1.13) into (3.5) and using (1.17) and (1.18), one ends up with

$$t_H^c = \alpha_H \tilde{t}^H + (1-\alpha_H)\tilde{t}^h, \qquad \tilde{t}^H \equiv \frac{\delta t^H + \left(1+t^H\right)\left(\tau^H - t^r r\right)}{\left(1+t^H\right)\left[r\left(1-t^r\right)+\tau^H+\delta\right]}, \tag{3.6}$$

$$\tilde{t}^h \equiv \frac{\delta t^H + \left(1+t^H\right)\tau^h}{\left(1+t^H\right)\left(r+\tau^h+\delta\right)}, \qquad \alpha_H \equiv \frac{\eta\left[r\left(1-t^r\right)+\tau^H+\delta\right]^{1-\sigma_h}}{\eta\left[r\left(1-t^r\right)+\tau^H+\delta\right]^{1-\sigma_h}+(1-\eta)\left(r+\tau^h+\delta\right)^{1-\sigma_h}}.$$

We see that the effective tax-inclusive ad valorem indirect tax rate on housing consumption is a weighted average of the tax-inclusive ad valorem tax rate on owner-occupied housing ($\tilde{t}^H$) and the tax-inclusive ad valorem tax rate on rental housing ($\tilde{t}^h$).

The effective indirect tax rate on total consumption (t^c) may now be derived from the constraint

$$t^c P C_i = t_H^c P_H C_{Hi} + t_o^c P_o C_{oi}, \qquad i = 1,2. \tag{3.7}$$

Given (3.2) and (3.5), we see that the aggregate indirect tax rate t^c satisfying (3.7) generates a total indirect tax revenue equal to the actual revenue collected. Substituting (1.6) through (1.8) into (3.7), we find that the aggregate indirect tax rate is a weighted average of the ad valorem tax rates on housing and on other consumption:

$$t^c = \beta_H t_H^c + (1-\beta_H) t_o^c, \qquad \beta_H \equiv \frac{\mu P_H^{1-\sigma}}{\mu P_H^{1-\sigma} + (1-\mu) P_o^{1-\sigma}}. \tag{3.8}$$

3.3 The effective aggregate investment tax wedge

Using a similar approach, we define the effective aggregate investment tax wedge on capital invested in closely held firms (t_c^k) which will ensure the same revenue as that generated by the actual investment tax wedges on the two organizational forms within the sector:

$$t_c^k K_c = t_{cc}^k K_{cc} + t_{cp}^k K_{cp}. \tag{3.9}$$

Inserting (2.26) and (2.27) into (3.9) and using (2.34) and (2.35), we find:

$$t_c^k = \chi \left(\frac{\rho_{cc}}{\rho_c} \right)^{-\sigma_c} \left(\rho_{cc} - \delta_k - r \right) + \left(1 - \chi \right) \left(\frac{\rho_{cp}}{\rho_c} \right)^{-\sigma_c} \left(\rho_{cp} - \delta_k - r \right). \tag{3.10}$$

The aggregate investment tax wedge on all business capital is specified in a parallel manner as:

$$t^k K = t_c^k K_c + t_w^k K_w. \tag{3.11}$$

Substituting (2.23) and (2.24) into (3.11), we obtain

$$t^k = \varpi \left(\frac{\rho_w}{\rho} \right)^{-\sigma_k} t_w^k + \left(1 - \varpi \right) \left(\frac{\rho_c}{\rho} \right)^{-\sigma_k} t_c^k, \tag{3.12}$$

where t_w^k and t_c^k are given by (2.33) and (3.10), respectively.

3.4 Government revenue revisited

We may now simplify the expression for the present value of the net revenue collected from each cohort. Using the household budget constraint (1.24) and the definition of total saving given in (1.26) along with (3.2), (3.5), (3.7), (3.9) and (3.11), we can rewrite (3.1) as

$$R = t^c \left(PC_1 + pPC_2 \right) + t^w WL - B_1 + t^k K + p\left(t^r rPS - B_2 \right) \tag{3.13}$$

$$= \left[t^w + t^c \left(1 - t^w \right) \right] WL - \left(1 - t^c \right) \left(B_1 + pB_2 \right) + t^k K + p\left(t^r rPS - B_2 \right), \quad p \equiv \frac{1}{1 + r\left(1 - t^r \right)}.$$

4 The marginal deadweight loss from taxation[3]

4.1 Measuring total and marginal deadweight loss

We are now ready to specify the deadweight loss from taxation. For this purpose we introduce the consumer's expenditure function measuring the minimum amount of (exogenous) income the consumer needs to achieve a given level of utility $\bar{U}$. In our context with the utility function (1.1) and the budget constraint (1.24), the expenditure function (E) is found by solving the problem:

$$\underset{\text{w.r.t. } C_1, C_2, L}{\text{Minimize}} \quad E \equiv PC_1 + pPC_2 - W\left(1-t^w\right)L \quad \text{subject to} \quad U\left(C_1, C_2, L\right) = \bar{U}. \tag{4.1}$$

The solution to this problem yields an expenditure function of the form $E = E\left(P_1, P_2, W\left(1-t^w\right), \bar{U}\right)$ where $P_1 \equiv P$ is the price of first-period consumption and $P_2 \equiv pP$ is the price of second-period consumption, while $W\left(1-t^w\right)$ is price (opportunity cost) of leisure. From standard duality theory, the derivatives of this expenditure function are

$$\frac{\partial E}{\partial W\left(1-t^w\right)} = -L, \tag{4.2}$$

$$\frac{\partial E}{\partial P} = \frac{\partial E}{\partial P_1} + p\frac{\partial E}{\partial P_2} = C_1 + pC_2, \tag{4.3}$$

$$\frac{\partial E}{\partial p} = P \cdot \frac{\partial E}{\partial P_2} = PC_2 = p^{-1}\left(PS + pB_2\right), \tag{4.4}$$

[3] The method of calculating deadweight loss presented in this section is inspired by Benge (1999), but whereas he only considers direct taxes on labour income and on business income, we also account for consumption taxes and taxes on savings income.

where the last equality in (4.4) follows from the consumer's second-period budget constraint $PC_2 = \left[1 + r\left(1 - t^r\right)\right]PS + B_2$.

Adopting an equivalent variation measure, we define the total deadweight loss from taxation as the difference between the maximum amount consumers would be willing to pay to get rid of all taxes (given the level of utility $\bar{U}$ prevailing after the imposition of taxes) and the actual tax revenue collected. Given our expenditure function $E = E\left(P, pP, W\left(1 - t^w\right), \bar{U}\right)$, the total deadweight loss from taxation (DWL) is therefore equal to:

$$DWL = \overbrace{E\left(P, pP, W\left(1 - t^w\right), \bar{U}\right) - E\left(P^p, \frac{P^p}{1 + r}, W, \bar{U}\right)}^{\text{Equivalent variation}} - R, \qquad (4.5)$$

where P^p is a constant producer price index which would equal the consumer price level in the absence of indirect taxes. Thus $E\left(P^p, \frac{P^p}{1+r}, W, \bar{U}\right)$ is the minimum (exogenous) income needed to attain the utility level $\bar{U}$ if there were no distorting taxes. Hence the 'excess burden' in (4.5) measures the additional revenue that could have been raised by a non-distortionary lump sum tax rather than through the existing distortionary taxes without leaving consumers worse off. The relative after-tax prices P, p and $W\left(1 - t^w\right)$ depend on the tax rates t^c, t^r, t^w and t^k (where the effect of t^k stems from its impact on W), so from (4.5) it follows that the *marginal* deadweight loss from an increase in the some tax rate t^i is

$$\frac{dDWL}{dt^i} = \frac{dE}{dt^i} - \frac{dR}{dt^i} = \frac{dE}{dt^i} - \left(\frac{dR^s}{dt^i} + \frac{dR^d}{dt^i}\right), \qquad i = c, k, r, w. \quad (4.6)$$

To obtain the last equality in (4.6), we have split the total revenue change dR into the 'static' revenue change dR^s that would occur if taxpayers did not change their behaviour, and the 'dynamic' revenue change dR^d resulting from the behavioural responses to the change in the tax rate. Note that the derivatives in (4.6) are calculated on the assumption that the taxpayer is compensated so as to maintain the given utility level $\bar{U}$ prevailing before the tax increase.[4] The dynamic revenue change dR^d therefore stems exclusively from the substitution effects induced by the tax change.

The static revenue gain dR^s is calculated by assuming that C_1, C_2 and L are unchanged. To maintain a clean distinction between static and dynamic revenue changes we also assume that the compensation paid out to consumers to preserve their utility level is distributed across the life cycle in a way that does not require any changes in first-period savings to keep C_1, C_2 and L constant.

The decomposition of the total revenue change into a "static" and a "dynamic" component will turn out to be extremely useful, since we generally have $dE/dt^i = dR^s/dt^i$, as we shall demonstrate in the sections below. In other words, the static revenue gain will be just sufficient to compensate taxpayers for the tax increase, so the marginal deadweight loss will equal the dynamic revenue loss from the behavioural responses to the tax change. The intuition for this important result may be explained as follows: If taxpayers did not change their behaviour, it is immediately clear that the amount needed to compensate them would equal the static revenue gain from the tax increase, since this compensation would keep disposable incomes unchanged and allow taxpayers to maintain the same levels of present and future consumption at the same level of labour supply as before. In reality taxpayers do change their behaviour since the tax increase faces them with a new set of relative prices, but the substitution effects induced by a small tax change have no first-order impact on taxpayer welfare. The reason is that taxpayers had optimized their consumption and labour supply before the tax change, so by definition they are indifferent to working and saving a little more or a little less (this is just an application of the Envelope Theorem). Because the behavioural responses to a small tax change have no first-order effect on the utility of taxpayers, the static revenue gain is still sufficient to compensate them. Hence the net deadweight loss to society equals the government's revenue loss from the substitution effects on labour supply and consumption (saving) induced by the tax increase. When the initial tax wedges are positive, these behavioural responses to a tax increase generate a negative "fiscal externality" on the public budget, resulting in a first-order social efficiency loss.

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[4] While the *total* deadweight loss in (4.5) is calculated by means of the equivalent variation, we are thus effectively adopting a compensating variation measure of the *marginal* deadweight loss.

To obtain a measure of the efficiency loss that is independent of the units in which income and revenue are measured, it is useful to express the marginal deadweight loss as a fraction of the static revenue gain. When doing so, we obtain the so-called degree of self-financing associated (*DSF*) with the tax instrument t^i:

$$DSF_{t^i} \equiv \frac{dDWL / dt^i}{dR^s / dt^i} = -\frac{dR^d / dt^i}{dR^s / dt^i}, \tag{4.7}$$

where we have used (4.6) and the result that $dE / dt^i = dR^s / dt^i$. The *DSF* measures the fraction of the initial revenue gain from a tax increase which is lost again due to behavioural responses. In the case of a decrease in some tax, the DSF indicates the degree to which the tax cut pays for itself through behavioural changes that increase the tax base. A positive marginal deadweight loss is thus equivalent to a positive degree of self-financing.

4.2 The marginal deadweight loss from a rise in the labour income tax rate

Given our assumption of a linear labour income tax, the marginal labour income tax rate t^w applies to the entire wage bill *WL*. In practice the labour income tax is rarely linear, but when analyzing the effect of a rise in t^w, we may think of our model as simulating the effect of an equiproportionate rise in the marginal tax rate for all income groups in the economy.

From (4.2) it follows that $\partial E / \partial t^w = WL$, so the amount needed to compensate taxpayers for a unit rise in t^w is *WL*. According to the first line in (3.13), the static revenue gain that would result from a unit rise in t^w if consumption and labour supply behaviour were unchanged is likewise equal to *WL*. From (4.6) we thus have

$$\frac{dDWL}{dt^w} = \frac{dE}{dt^w} - \left(\frac{dR^s}{dt^w} + \frac{dR^d}{dt^w} \right) = WL - \left(WL + \frac{dR^d}{dt^w} \right) = -\frac{dR^d}{dt^w}. \tag{4.8}$$

This confirms the general result reported above, i.e., that $dDWL / dt^i = -dR^d / dt^i$. To calculate the dynamic revenue change induced by a rise in the marginal labour income tax rate, we use the result in (2.6) that a change in labour supply will change the stock of business capital by the amount $dK = (K/L)\cdot dL$. Since $w \equiv W(1-t^w)/P$, a rise in t^w reduces the real after-tax consumer wage by the amount $dw = -(w/(1-t^w))\cdot dt^w$. From the second line in (3.13) it then follows that:

$$\frac{dR^d}{dt^w} = -\left(\frac{w}{1-t^w}\right)\left\{\left[t^w + t^c\left(1-t^w\right)\right]W\frac{\partial L}{\partial w} + t^k\frac{K}{L}\frac{\partial L}{\partial w} + pt^r rP\frac{\partial S}{\partial w}\right\}. \quad (4.9)$$

Rewriting (4.9) in terms of elasticities and expressing the result as a fraction of the static revenue gain *WL*, we obtain the degree of self-financing associated with a small change in the marginal labour income tax rate:

$$\frac{dDWL/dt^w}{dR^s/dt^w} = -\frac{dR^d/dt^w}{dR^s/dt^w} = \frac{\left(m^w + m^k\theta^k\right)\varepsilon_w^L + pt^r\theta^s\varepsilon_w^S}{1-t^w}, \quad (4.10)$$

$$\varepsilon_w^L \equiv \frac{\partial L}{\partial w}\frac{w}{L},\quad \varepsilon_w^S \equiv \frac{\partial S}{\partial w}\frac{w}{S},\quad m^w \equiv t^w + t^c\left(1-t^w\right),\quad m^k \equiv \frac{t^k}{\rho-\delta_k},\quad \theta^k \equiv \frac{(\rho-\delta_k)K}{WL},\quad \theta^s \equiv \frac{rPS}{WL}.$$

The variable m^w is the total marginal effective tax rate on labour income, including the indirect taxes that work in part like a tax on labour income by eroding the real consumer wage. The variable m^k is the marginal effective investment tax rate, expressing the investment tax wedge as a fraction of the pre-tax return $\rho-\delta_k$ on the marginal investment. In addition, (4.10) includes the parameters θ^k and θ^s indicating the importance of investment and savings income relative to wage income. Recall that since our measure of deadweight loss assumes that consumers are compensated for the tax increase, the wage elasticities of labour supply and savings in (4.10) (ε_w^L and ε_w^S) are *compensated* elasticities. If the marginal savings rate equals the average savings rate *s*, there is a simple link between the two elasticities. By definition, we have

$$S = s\cdot\left(wL + \frac{B_1}{P}\right). \quad (4.11)$$

Compensation implies that the direct impact of a rise in t^w on real after-tax labour income wL is offset by a corresponding rise in the real benefit B_1/P. Hence the compensated change in savings is driven solely by the compensated change in labour supply, so from (4.11) and the assumption of a constant savings rate we get:

$$dS = s \cdot wdL \quad \Rightarrow \quad \frac{dS}{dw} = sL\frac{dL}{dw}\frac{w}{L} \quad \Rightarrow \quad \varepsilon_w^S = \frac{swL}{S} \cdot \varepsilon_w^L. \qquad (4.12)$$

Inserting (4.11) into (4.12) and dividing by , we obtain:

$$\varepsilon_w^S = \left(\frac{1-t^w}{1-t^w+b_1}\right)\varepsilon_w^L, \qquad b_1 \equiv \frac{B_1}{WL}. \qquad (4.13)$$

Using the definition of m^w, the total deadweight loss in (4.10) may be decomposed into the dynamic revenue losses from the decline in the four tax bases considered:

$$\frac{dDWL/dt^w}{dR^s/dt^w} = \overbrace{\left(\frac{t^w\varepsilon_w^L}{1-t^w}\right)}^{\text{loss of labour income tax revenue}} + \overbrace{\left(\frac{t^c\left(1-t^w\right)\varepsilon_w^L}{1-t^w}\right)}^{\text{loss of consumption tax revenue}} + \overbrace{\left(\frac{m^k\theta^k\varepsilon_w^L}{1-t^w}\right)}^{\text{loss of business income tax revenue}} + \overbrace{\left(\frac{pt^r\theta^s\varepsilon_w^S}{1-t^w}\right)}^{\text{loss of savings tax revenue}}. \qquad (4.14)$$

4.3 The marginal deadweight loss from a rise in the consumption tax rate

Consider next the effect of a rise in the consumption tax rate. Since the consumer price index P includes indirect taxes amounting to t^cP, we have $\partial P/\partial t^c = P$. It then follows from (4.3) that the amount needed to compensate consumers for a unit rise in t^c is $PC_1 + pPC_2$. Now suppose that consumers are compensated for the tax increase by a rise in the transfer B_1 equal to the amount PC_1 and by a rise in B_2 amounting to PC_2, so that the present value of the rise in B_2 equals pPC_2. In total, these increases in transfers are just sufficient to fully compensate consumers. Further, since the increases in B_1 and B_2 exactly compensate for the rise in the tax-inclusive consumption expenditure in each of the two periods (given the initial consumption levels), the consumer does not have to change his first-period saving PS to maintain constant levels of C_1, C_2 and L.[5] From these observations it follows from (4.3), (4.6) and the first line in (3.13) that

$$\frac{dDWL}{dt^c}=\frac{dE}{dt^c}-\left(\frac{dR^s}{dt^c}+\frac{dR^d}{dt^c}\right)=P\left(C_1+pC_2\right)-\left(P\left(C_1+pC_2\right)+\frac{dR^d}{dt^c}\right)=-\frac{dR^d}{dt^c}. \quad (4.15)$$

The rise in the consumption tax rate induces behavioural changes through its impact on the real after-tax consumer wage $w\equiv W\left(1-t^w\right)/P$. Recalling that $\partial P/\partial t^c=P$, we have $\partial w/\partial t^c=-w$, and from (2.6) we still have $dK=\left(K/L\right)\cdot dL$. Using the second line in (3.13) and the definitions stated in (4.10), we then find:

$$\frac{dR^d}{dt^c}=-WL\left[\left(m^w+m^k\theta^k\right)\varepsilon_w^L+pt^r\theta^s\varepsilon_w^S\right]. \quad (4.16)$$

The second line in (3.13) also implies that the static revenue gain is:

$$\frac{dR^s}{dt^c}=\left(1-t^w\right)WL+B_1+pB_2. \quad (4.17)$$

From (4.10), (4.13) and (4.14) it follows that

$$\frac{dDWL/dt^c}{dR^s/dt^c}=-\frac{dR^d/dt^c}{dR^s/dt^c}=\frac{\left(m^w+m^k\theta^k\right)\varepsilon_w^L+pt^r\theta^s\varepsilon_w^S}{1-t^w+b_1+pb_2}$$

$$=\left(\frac{1-t^w}{1-t^w+b_1+pb_2}\right)\frac{dDWL/dt^w}{dR^s/dt^w}, \qquad b_1\equiv\frac{B_1}{WL}, \qquad b_2\equiv\frac{B_2}{WL}, \quad (4.18)$$

where b_1 and b_2 are the replacement rates in the public transfer system for the young and the old, respectively. We see from the second line in (4.15) that the deadweight loss from a rise in the consumption tax rate is lower than the deadweight loss from a rise in the labour income tax rate involving the same static revenue gain. The reason is that the consumption tax is levied on a broader base which includes the consumption financed out of the public

[5] The consumer's second-period budget constraint is $PS=p\left(PC_2-B_2\right)$. At the initial level of C_2, a unit rise in t^c raises PC_2 by the amount $\left(\partial P/\partial t^c\right)\cdot C_2=PC_2$, but since B_2 goes up by a similar amount, PS is unchanged. A similar conclusion follows from the first-period budget constraint $PS=\left(1-t^w\right)WL+B_1-PC_1$, since $dB_1=PC_1=\left(\partial P/\partial t^c\right)\cdot C_1$.

transfers B_1 and B_2. Since these income components are exogenous to consumers, the consumption tax imposed on them is effectively a non-distorting lump-sum tax.

4.4 The marginal deadweight loss from a rise in the investment tax wedge

A rise in the effective source-based capital tax rate t^k (the investment tax wedge) could be implemented through a rise in the statutory corporate income tax rate or via measures to broaden the business income tax base. We focus here on a change in the investment tax wedges t_w, t_{cc} and t_{cp} inducing an equi-proportionate rise in the costs of capital ρ_w, ρ_{cc} and ρ_{cp} for the three organizational forms considered.[6] From (2.25), (2.28), (3.10) and (3.12) it is easy to show that such a change in the business income tax code implies that $\partial\rho/\partial t^k = 1$. According to (2.14) the burden of this rise in the cost of capital will be shifted to domestic workers through a fall in the wage rate equal to $dW/\partial t^k = -(K/L)$. Faced with this drop in the pre-tax wage rate, we see from (4.2) that workers will have to be compensated by the amount $-(1-t^w)L\cdot(\partial W/\partial t^k) = -(1-t^w)K$ to maintain the same utility level. We also see from the first line in (3.13) that the static revenue gain from a unit rise in t^k will be $dR^s = K + t^w L\cdot(\partial W/\partial t^t) = K(1-t^w)$, assuming that workers are compensated through a rise in B_1 so that their savings PS can be kept unchanged as long as C_1, C_2 and L are unchanged. Thus we have

$$\frac{dDWL}{dt^k} = \frac{dE}{dt^k} - \left(\frac{dR^s}{dt^k} + \frac{dR^d}{dt^k}\right) = (1-t^w)K - \left((1-t^w)K + \frac{dR^d}{dt^k}\right) = -\frac{dR^d}{dt^k}. \qquad (4.19)$$

Using the second line in (3.13) and the definitions in (4.10) plus the fact that $d\rho = dt^k$, we find the dynamic revenue effect to be

$$\frac{dR^d}{dt^k} = -K\left[m^k \tfrac{c}{\rho}\varepsilon_\rho^K + (m^w + \theta^k m^k)\varepsilon_w^L + pt^r\theta^s\varepsilon_w^S\right], \qquad \varepsilon_\rho^K \equiv -\frac{\partial K}{\partial\rho}\frac{\rho}{K} > 0, \qquad (4.20)$$

where we recall from (2.18) that the elasticity ε_ρ^K is calculated at the initial level of labour supply. Dividing (4.17) this by the static revenue gain $dR^s/dt^k = (1-t^w)K$ and using (4.10), we get

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[6] In section 5 we shall consider the efficiency costs of non-uniform business taxation.

$$\frac{dDWL/dt^k}{dR^s/dt^k} = -\frac{dR^d/dt^k}{dR^s/dt^k} = \frac{m^k \frac{c}{\rho}\varepsilon_\rho^K + \left(m^w + \theta^k m^k\right)\varepsilon_w^L + pt^r\theta^s\varepsilon_w^S}{1-t^w}$$

$$= \frac{m^k \frac{c}{\rho}\varepsilon_\rho^K}{1-t^w} + \frac{dDWL/dt^w}{dR^s/dt^w}. \qquad (4.21)$$

We see that the deadweight loss from a higher investment tax wedge consists of the revenue loss from the direct negative impact on domestic investment – represented by the first term in the second line in (4.18) – plus the revenue loss arising as the fall in wages induced by lower investment reduces labour supply and savings. Note that since the direct negative impact on investment does not arise under a labour income tax which therefore generates a lower deadweight loss per unit of revenue than a tax on domestic investment, as shown by the second line in (4.18). This is just an illustration of the well-known result that it is inoptimal to impose a source-based tax on the normal return to investment in a small open economy.

By analogy to (4.11), we can decompose the marginal deadweight loss into the losses stemming from the shrinkage of the various tax bases:

$$\frac{dDWL/dt^k}{dR^s/dt^k} = \overbrace{\left(\frac{t^w\varepsilon_w^L}{1-t^w}\right)}^{\text{loss of labour income tax revenue}} + \overbrace{\left(\frac{t^c\left(1-t^w\right)\varepsilon_w^L}{1-t^w}\right)}^{\text{loss of consumption tax revenue}} + \overbrace{\left(\frac{m^k\left(\frac{c}{\rho}\varepsilon_\rho^K + \theta^k\varepsilon_w^L\right)}{1-t^w}\right)}^{\text{loss of business income tax revenue}} + \overbrace{\left(\frac{pt^r\theta^s\varepsilon_w^S}{1-t^w}\right)}^{\text{loss of savings tax revenue}}. \qquad (4.22)$$

4.5 The marginal deadweight loss from a rise in the savings tax wedge

We finally consider the effects of a rise in the residence-based capital income tax rate t^r. We assume that the tax rate applies to returns to equity as well as to interest income ($t^e = t^r$), so according to (2.35) the rise in t^r will have no impact on the cost of equity finance for closely held firms and hence no effect on the user cost of capital. As a consequence, there will be no effect on wages either, so only the relative price of future consumption (p) will be affected. Since $p \equiv 1/\left[1+r\left(1-t^{r}\right)\right]$, we find from (4.4) that consumers would need to be compensated by the amount

$$\frac{dE}{dt^{r}} = \frac{\partial p}{\partial t^{r}} \cdot \frac{\partial E}{\partial p} = rp^{2}PC_{2} = rp\left(PS + pB_{2}\right), \tag{4.23}$$

for a unit rise in t^{r}. This compensation could be achieved by raising B_2 by the amount $dB_2 = rp(PC_2 - B_2) \cdot dt^{r}$, since the present value of the second-period government transfer would then change by

$$\frac{d\left(pB_2\right)}{dt^{r}} = \frac{\partial p}{\partial t^{r}} B_2 + \frac{\partial B_2}{\partial t^{r}} p = rp^{2}B_2 + rp^{2}\left(PC_2 - B_2\right) = rp^{2}PC_2 = rp\left(PS + pB_2\right). \tag{4.24}$$

Note that since the consumer's second-period budget constraint requires $PS = pPC_2 - pB_2$, the compensating rise in B_2 will keep total savings constant as long as C_2 is constant, since constancy of C_2 implies that $\partial(pPC_2)/\partial t^{r} = rp^{2}PC_2 = \partial(pB_2)/\partial t^{r}$, where the last equality follows from (4.24).

In the absence of behavioural responses, the marginal deadweight loss from the tax increase equals the present value of the additional public transfer needed to compensate consumers, dE/dt^{r}, minus the static revenue gain from the higher tax on savings income, $d(pt^{r}rPS)/dt^{r}$. Using (4.20), (4.21) and the definition of *p*, as well as the fact that *PS* is constant in the absence of behavioural responses, we thus have

$$\begin{aligned} \frac{dE}{dt^{r}} - \frac{dR^{s}}{dt^{r}} &= \frac{d\left(pB_2\right)}{dt^{r}} - \frac{d\left(pt^{r}rPS\right)}{dt^{r}} = rp\left(PS + pB_2\right) - \left(rpPS + r^{2}p^{2}t^{r}PS\right) \\ &= rp^{2}\left(B_2 - t^{r}rPS\right) \end{aligned} \tag{4.25}$$

When taxes and transfers are zero in the initial equilibrium ($t^{r} = B_2 = 0$), (4.25) reproduces the standard result that the compensating variation (dE) equals the mechanical revenue gain. With positive initial taxes and transfers, the sign of $dE/dt^{r} - dR^{s}/dt^{r}$ can go either way, according to (4.25). However, it is natural to focus on the benchmark case where the revenue from capital income taxes ($t^{r}rPS$) corresponds to the public transfers to the old (B_2). In the following, we will thus assume that $t^r rPS = B_2$.[7] From (4.6) and (4.25) we then get the standard result that the marginal deadweight loss equals the revenue loss from the behavioural responses to the tax increase, that is, $dDWL/dt^r = -dR^d/dt^r$.

Let $r^a \equiv r(1-t^r)$ denote the after-tax real interest rate, implying $dr^a/dt^r = -r$. From the second line in (3.13) and the fact that $dK = (K/L)\cdot dL$ as long as the cost of capital is constant, we then find

$$\frac{dR^d}{dt^r} = -r\left(m^w W\frac{\partial L}{\partial r^a} + t^k\frac{K}{L}\frac{\partial L}{\partial r^a} + pt^r rP\frac{\partial S}{\partial r^a}\right) \quad \Leftrightarrow$$

$$\frac{dR^d}{dt^r} = -\left(\frac{WL}{1-t^r}\right)\left[\left(m^w + \theta^k m^k\right)\varepsilon_r^L + pt^r\theta^s\varepsilon_r^S\right], \quad \varepsilon_r^L \equiv \frac{\partial L}{\partial r^a}\frac{r^a}{L}, \quad \varepsilon_r^S \equiv \frac{\partial S}{\partial r^a}\frac{r^a}{S}. \qquad (4.26)$$

As shown above, the static revenue gain equals $dR^s/dt^r = rp(PS + pB_2)$ under our benchmark assumption that $B_2 - t^r rPS$. Hence we get

$$\frac{dDWL/dt^r}{dR^s/dt^r} = -\frac{dR^d/dt^r}{dR^s/dt^r} = \frac{p^{-1}\left(m^w + \theta^k m^k\right)\varepsilon_r^L + t^r\theta^s\varepsilon_r^S}{\left(1-t^r\right)\left(\theta^s + rpb_2\right)}, \qquad (4.27)$$

which may be decomposed as follows:

$$\frac{dDWL/dt^r}{dR^s/dt^r} = \overbrace{\left(\frac{p^{-1}t^w\varepsilon_r^L}{\left(1-t^r\right)\left(\theta^s + rpb_2\right)}\right)}^{\text{loss of labour income tax revenue}} + \overbrace{\left(\frac{p^{-1}t^c\left(1-t^w\right)\varepsilon_r^L}{\left(1-t^r\right)\left(\theta^s + rpb_2\right)}\right)}^{\text{loss of consumption tax revenue}}$$
$$+ \overbrace{\left(\frac{p^{-1}\theta^k m^k\varepsilon_r^L}{\left(1-t^r\right)\left(\theta^s + rpb_2\right)}\right)}^{\text{loss of business income tax revenue}} + \overbrace{\left(\frac{t^r\theta^s\varepsilon_r^S}{\left(1-t^r\right)\left(\theta^s + rpb_2\right)}\right)}^{\text{loss of savings tax revenue}} \qquad (4.28)$$

[7] This assumption is unlikely to hold in practice, but the approximation error from adopting it is probably minor, due to the 'double discounting' of the last term in (4.25): the realistic length of a time period in our model is 25-30 years, so the discount factor $p \equiv 1/\left[1 + r\left(1-t^r\right)\right]$ will be far below unity.

The first three terms on the right-hand side of (4.25) include the compensated real interest elasticity of labour supply, ε_r^L, about which little is known. However, as shown in Appendix 2, the compensated interest elasticity of labour supply is linked to the compensated wage elasticity of savings by the following formula, where s is the average savings rate during the first period of the representative consumer's life cycle:

$$\varepsilon_r^L = s \cdot \left(\frac{r^a}{1+r^a} \right) \cdot \varepsilon_w^L. \tag{4.29}$$

Appendix 2 also shows that the compensated elasticity of savings with respect to the after-tax real interest rate is linked to the corresponding *uncompensated* elasticity ($\hat{\varepsilon}_r^S$) via the formula

$$\varepsilon_r^S = \hat{\varepsilon}_r^S + \left(\frac{r^a}{1+r^a} \right) (1+b) \left(1 - c_Y \varepsilon_I^C \right), \tag{4.30}$$

$$b \equiv \frac{pb_2}{s\left(1-t^w+b_1\right)}, \qquad Y \equiv \frac{W\left(1-t^w\right)+B_1+pB_2}{P}, \qquad c_Y \equiv \frac{pC_2}{Y}, \qquad \varepsilon_Y^C \equiv \frac{\partial C_2}{\partial Y}\frac{Y}{C_2},$$

where b measures the amount of old-age consumption which is financed by public pensions relative to consumption financed by previous savings, Y is the present value of potential lifetime income, c_Y is the share of potential lifetime income spent on consumption in retirement, and ε_Y^C is the income elasticity of demand for consumption in retirement. Equation (4.30) may be useful for calibrating a realistic value of the compensated interest elasticity of saving.

5 The deadweight loss from non-uniform taxation

This main section derives a set of formulas which may be used to estimate the deadweight loss from non-uniform taxation across different types of investment and consumption. The general approach is to calculate the additional revenue that could be gained through a switch to uniform taxation without reducing consumer welfare.

5.1 The deadweight loss from tax distortions to the choice of organizational form

A non-neutral taxation of widely and closely held firms will generate differences in the user cost of capital across these alternative forms of business organization. As we shall now show, a switch to uniform taxation ensuring an identical cost of capital across organizational forms would allow the government to collect additional revenue without increasing the overall cost of business capital. As a consequence, aggregate investment and the average real wage would be unaffected, and hence consumer welfare would also remain unchanged. The deadweight loss from non-uniform taxation across organizational forms may thus be measured by the additional revenue that could be gained by equalizing the cost of capital for the two organizational forms at the level of the current aggregate cost of business capital. From (2.25) and the facts that $d\rho_w = dt_w^k$ and $d\rho_c = dt_c^k$, it follows that such a tax reform will have to satisfy

$$d\rho = 0 \quad \Rightarrow \quad \varpi \rho_w^{-\sigma_k} \cdot d\rho_w + (1-\varpi)\,\rho_c^{-\sigma_k} \cdot d\rho_c = 0 \quad \Leftrightarrow$$

$$dt_c^k = -\left(\frac{\varpi}{1-\varpi}\right)\left(\frac{\rho_c}{\rho_w}\right)^{\sigma_k} dt_w^k. \tag{5.1}$$

Since the total revenue from business income taxes is $R_k \equiv t_w^k K_w + t_c^k K_c$, the change in business income tax revenue generated by the reform is

$$dR_k = \overbrace{dt_w^k \cdot K_w + dt_c^k \cdot K_c}^{\text{static revenue effect}} + \overbrace{t_w^k \cdot \frac{dK_w}{d\rho_w} \cdot dt_w^k + t_c^k \cdot \frac{dK_c}{d\rho_c} \cdot dt_c^k}^{\text{dynamic revenue effect}}. \tag{5.2}$$

From (5.1) it is easy to show that the static revenue effect in (5.2) is zero. Using (3.11) plus (2.23) and (2.24), and recalling that (5.1) ensures that ρ and hence K stay constant, we can write the dynamic revenue effect in (5.2) as

$$dR_k = -\sigma_k \rho^{\sigma_k} K \left[t_w^k \varpi \rho^{-(\sigma_k+1)} d\rho_w + t_c^k (1-\varpi)\,\rho^{-(\sigma_k+1)} d\rho_c \right]. \tag{5.3}$$

Inserting (5.1) into (5.3), one finds after some manipulations that

$$\frac{dR_K}{\rho K} = \varpi \sigma_k \left(\frac{\rho}{\rho_w}\right)^{\sigma_k} \left(\frac{t_c^k}{\rho_c} - \frac{t_w^k}{\rho_w}\right) \frac{d\rho_w}{\rho}. \tag{5.4}$$

An equalization of the effective investment tax wedges t_c^k and t_w^k and hence an equalization of ρ_c and ρ_w at a level ensuring a constant value of ρ implies that $d\rho_w = -(\rho_w - \rho)$ so that (5.4) becomes

$$\frac{dR^K}{\rho K} = \varpi \sigma_k \left(\frac{\rho}{\rho_w}\right)^{\sigma_k} \left(\frac{t_w^k}{\rho_w} - \frac{t_c^k}{\rho_c}\right) \frac{(\rho_w - \rho)}{\rho} > 0. \tag{5.5}$$

Equation (5.5) captures the dynamic revenue gain emerging as the tax reform induces substitution towards the organizational form which is more heavily taxed at the outset. The positive sign of the expression on the right-hand side of (5.5) follows from the fact that the last two brackets will always have the same sign, that is, if $t_w^k / \rho_w > t_c^k / \rho_c$ we must have $\rho_w > \rho$, and vice versa.

Following a similar procedure, one can show that an equalization of the tax wedges on closely held incorporated and unincorporated firms at a level ensuring a constant value of ρ_c will generate a revenue gain relative to gross pre-tax profits equal to

$$\frac{dR_c^K}{\rho_c K_c} = \chi\sigma_c \left(\frac{\rho_c}{\rho_{cc}}\right)^{\sigma_c} \left(\frac{t_{cc}^k}{\rho_{cc}} - \frac{t_{cp}^k}{\rho_{cp}}\right) \frac{(\rho_{cc} - \rho_c)}{\rho_c} > 0. \tag{5.6}$$

5.2 The deadweight loss from tax distortions to portfolio composition

In this section we show that a switch to uniform taxation of the return to alternative forms of financial saving would enable the government to collect additional tax revenue without reducing the overall return to saving and hence without reducing consumer welfare. The additional revenue is therefore a measure of the efficiency loss from non-neutral taxation of savings.

From (1.30) and (1.31) it follows that the total revenue from taxes on the return to financial saving is

$$R^S = t^I rS^I + t^F rS^F = \left[\gamma t^I \left(\frac{1-t^I}{1-t^r}\right)^{\phi} + (1-\gamma)t^F \left(\frac{1-t^F}{1-t^r}\right)^{\phi}\right] rS^f. \tag{5.7}$$

Assume now that the effective tax rates t^I and t^F are equalized at a level equal to the initial aggregate effective capital income tax rate t^r so that the overall after-tax return to saving $r(1-t^r)$ - and hence aggregate financial saving S^f - are unchanged. According to (1.32) we then have

$$dt^r = 0 \quad \Rightarrow \quad \gamma(1-t^I)^{\phi} dt^I + (1-\gamma)(1-t^F)^{\phi} dt^F = 0 \quad \Leftrightarrow$$

$$dt^F = -\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{1-t^I}{1-t^F}\right)^{\phi} \cdot dt^I. \tag{5.8}$$

Since (5.8) ensures constancy of t^r and S^f, equation (5.7) implies

$$\frac{dR^S}{rS^f} = \left(1-t^r\right)^{-\phi}\left\{\gamma\left[\left(1-t^I\right)^{\phi} - \phi t^I\left(1-t^I\right)^{\phi-1}\right]dt^I + \left(1-\gamma\right)\left[\left(1-t^F\right)^{\phi} - \phi t^F\left(1-t^F\right)^{\phi-1}\right]dt^F\right\}. \quad (5.9)$$

Substituting (5.8) into (5.9) and rearranging, and using the fact that $dt^I = t^r - t^I$, we get

$$\frac{dR^S}{rS^f} = \gamma\phi\left(\frac{1-t^I}{1-t^r}\right)^{\phi}\left(\frac{t^F}{1-t^F} - \frac{t^I}{1-t^I}\right)\left(t^r - t^I\right) > 0, \quad (5.10)$$

where the positive sign follows from (1.32) which implies that if $t^F > t^I$ we have $t^r > t^I$, and vice versa.

5.3 The deadweight loss from tax distortions to the pattern of consumption

The specification of consumer preferences in section 1.1 assumes that the utility function is homothetic and weakly separable in consumption and labour (leisure). It is well known from the theory of taxation that, in the absence of consumption externalities, the optimal tax policy in this case involves uniform indirect taxation. We start this section by deriving two alternative measures of the effiency loss from differentiation of the VAT, taking excise tax rates as given. The first measure treats the excises as ordinary indirect taxes, thus abstracting from the fact that excises may serve to internalize negative externalities. The second measure of deadweight loss goes to the other extreme by assuming that all excises are set at their 'correct' Pigovian level, reflecting the marginal external cost associated with consuming the good in question. In that case the excises are entirely non-distorting and may simply be ignored.

For the moment we focus on commodities other than housing, and we start by ignoring externalities from the consumption of goods subject to excises. In line with common practice in most countries, we assume that the VAT is levied on the value of sales *including* excise taxes. The total tax-exclusive indirect tax rate on goods of category n then becomes

$$t^n = t_e^n + t_v^n\left(1+t_e^n\right), \quad (5.11)$$

where t_v^n is the VAT rate, and t_e^n is the excise tax rate in case the goods category in question is also subject to excise taxation. A given goods category n includes all goods and services subject to the same VAT rate, so the total number N of different goods categories corresponds to the number of different VAT rates, including the groups of commodities subject to zero-rating (where the VAT on inputs is fully refunded) and to exemption (where the input VAT is not refunded). Consider a situation where some goods are initially subject to the standard VAT rate t_v while others are subject to reduced rates. Suppose then that the VAT rate on all goods and services is equalized at the level αt_v which is chosen such that the general consumer price level – and hence the level of consumer welfare – is kept constant. Since the excise tax rates are unaffected, it follows from (5.11) that the change in the overall indirect tax rate on goods category n is

$$dt^n = t_e^n + \alpha t_v\left(1+t_e^n\right) - \left[t_e^n + t_v^n\left(1+t_e^n\right)\right] = \left(\alpha t_v - t_v^n\right)\left(1+t_e^n\right). \quad (5.12)$$

The general consumer price index for goods other than housing is given by (1.15). To keep this price index constant, the parameter α in (5.12) must be chosen such that

$$dP_o = \sum_{n=1}^{N} \beta_n p_n^{-\sigma_o} \cdot dp_n = 0. \quad (5.13)$$

From (1.16) and (5.12) we have

$$p_n = 1+t^n \quad \Rightarrow \quad dp_n = dt^n = \left(\alpha t_v - t_v^n\right)\left(1+t_e^n\right). \quad (5.14)$$

Inserting (5.14) into (5.13) and solving for α , we get

$$\alpha = \frac{\sum_n \beta_n \left(1+t^n\right)^{-\sigma_o} t_v^n \left(1+t_e^n\right)}{t_v \sum_n \beta_n \left(1+t^n\right)^{-\sigma_o} \left(1+t_e^n\right)}. \quad (5.15)$$

Using (1.14), (1.15) and the facts that $p_n = 1+t^n$ and $dp_n = dt^n$, we obtain the following expression for the revenue change (dR_o) induced by the move towards a uniform VAT:

$$dR_o = \sum_n \left\{ dt^n \cdot x_n + t^n \cdot dx_n \right\} = \sum_n \left\{ dt^n x_n \left[1 + \frac{t^n}{p_n} \frac{dx_n}{dp_n} \frac{p_n}{x_n} \right] \right\} \quad \Rightarrow$$

$$\frac{dR_o}{P_o C_o} = \overbrace{\sum_n \beta_n \left(\frac{p_n}{P_o} \right)^{1-\sigma_o} \left(\frac{dt^n}{1+t^n} \right)}^{\text{static revenue effect}} \overbrace{- \sigma_o \sum_n \beta_n \left(\frac{p_n}{P_o} \right)^{1-\sigma_o} \left(\frac{dt^n}{1+t^n} \right) \left(\frac{t^n}{p_n} \right)}^{\text{dynamic revenue effect}}, \quad (5.16)$$

where dt^n can be found from (5.14) and (5.15). Since (5.13) and (5.14) imply that

$$\sum_n \beta_n p_n^{1-\sigma_o} \left(\frac{dt^n}{1+t^n} \right) = 0,$$

it follows that the static revenue effect indicated in (5.16) is also zero, so once again we find that the efficiency gain from the tax reform is given by the dynamic revenue effect reflecting consumer substitution towards the goods that become less heavily taxed.

The procedure above ignores non-fiscal externalities. Alternatively, one may assume that excise taxes serve to perfectly internalize the negative externalities from the consumption of the relevant goods. In that case the change in the revenue from excises on any goods category should not be included in our measure of the efficiency gain from tax reform, since this revenue change will be offset by a corresponding change in external costs, assuming that marginal external costs are constant and equal to the relevant VAT-inclusive excise tax rate, $t_e^n \left(1+t_v^n\right)$. Thus a unit increase in the consumption of the good x_n will generate a revenue gain equal to $t^n = t_v^n + t_e^n \left(1+t_v^n\right)$, representing a positive "fiscal externality", but it also generates a non-fiscal marginal externality cost equal to the excise component $t_e^n \left(1+t_v^n\right)$, so the net efficiency gain per unit of additional consumption is only $t^n - t_e^n \left(1+t_v^n\right) = t_v^n$. Hence we must replace the tax rate t^n by t_v^n in the numerator of the last term on the right-hand side of (5.16) to obtain a measure of the net efficiency gain from the tax reform.

A similar method may be used to derive the deadweight loss from non-uniform taxation of owner-occupied and rental housing. To illustrate, consider a property tax reform which equalizes the total taxation and hence the total user cost of the two forms of housing consumption. If $\hat{\tau}$ is the post-reform tax rate on rental property, the post-reform property tax rate on owner-occupied housing will have to be $\hat{\tau}+t^r r$ to offset the user cost reduction caused by the taxation and deductibility of interest under the personal income tax. After the reform, we then have

$$p_H = p_h = \left(1+t^H\right)\left(r+\hat{\tau}+\delta\right). \tag{5.17}$$

The value of $\hat{\tau}$ is chosen in a way that keeps the overall consumer price of housing services constant. From (1.13) it then follows that

$$dP_H = 0 \quad \Rightarrow \quad \eta p_H^{-\sigma_h} dp_H + \left(1-\eta\right) p_h^{-\sigma_h} dp_h = 0 \quad \Leftrightarrow$$

$$dp_h = -\left(\frac{\eta}{1-\eta}\right)\left(\frac{p_h}{p_H}\right)^{\sigma_h} dp_H. \tag{5.18}$$

From (1.17), (1.18) and (5.17) we find that the changes in user costs generated by the reform are

$$dp_h = \left(1+t^H\right)\left(\hat{\tau}-\tau^h\right) \qquad \text{and} \qquad dp_H = \left(1+t^H\right)\left(\hat{\tau}+t^r r-\tau^H\right), \tag{5.19}$$

which may be inserted into (5.18) to give

$$\hat{\tau} = \frac{\tau^h + \left(\tau^H - t^r r\right)\left(\frac{\eta}{1-\eta}\right)\left(\frac{p_h}{p_H}\right)^{\sigma_h}}{1+\left(\frac{\eta}{1-\eta}\right)\left(\frac{p_h}{p_H}\right)^{\sigma_h}}. \tag{5.20}$$

The user cost changes in (5.19) are identical to the changes in the effective tax rates per unit of the two forms of housing. Hence the change in revenue induced by the reform (dR_H) may be written as

$$dR_H = \overbrace{dp_H H + dp_h h}^{\text{static revenue effect}} + \overbrace{\left[t^H\delta+\left(1+t^H\right)\left(\tau^H - t^r r\right)\right]\frac{dH}{dp_H}dp_H + \left[t^H\delta+\left(1+t^H\right)\tau^h\right]\frac{dh}{dp_h}dp_h}^{\text{dynamic revenue effect}}. \tag{5.21}$$

From (5.18) one can show that the static revenue effect in (5.21) is zero. Using (1.11), (1.12) and (3.6), the dynamic revenue effect may be written as

$$dR_H = -\sigma_h \left(\tilde{t}^H H \cdot dp_H + \tilde{t}^h h \cdot dp_h \right). \tag{5.22}$$

Inserting (1.11), (1.12), (5.18) and the last equation in (5.19) into (5.22), one ends up with the following expression for the dynamic (and hence the total) revenue effect of the switch towards uniform housing taxation:

$$\begin{aligned} \frac{dR_H}{P_H C_H} &= \sigma_h \eta \left(\frac{p_H}{P_H} \right)^{1-\sigma_h} \left(\tilde{t}^h - \tilde{t}^H \right) \left(\frac{dp_H}{p_H} \right) \\ &= \sigma_h \eta \left(\frac{p_H}{P_H} \right)^{1-\sigma_h} \left(\tilde{t}^h - \tilde{t}^H \right) \left(\frac{\hat{\tau} + t^r r - \tau^H}{r\left(1-t^r\right) + \tau^H + \delta} \right) > 0. \end{aligned} \tag{5.23}$$

The term $\hat{\tau} + t^r r$ is the post-reform property tax rate on owner-occupied housing. If owner-occupied housing is favoured by the tax system prior to the property tax reform ($\tilde{t}^h > \tilde{t}^H$), the reform implies that $\hat{\tau} + t^r r > \tau^H$, whereas if $\tilde{t}^h < \tilde{t}^H$ before the reform, we have $\hat{\tau} + t^r r < \tau^H$, so when the initial tax regime involves non-uniform taxation, the reform will always generate a positive revenue gain, as indicated in (5.23).

We may also measure the efficiency loss from non-uniform taxation of housing consumption and other consumption. Without loss of generality, we may assume that the pre-tax value of all housing services ($r + \delta$) is initially subject to a uniform tax at the rate $\overline{t}^H$, as long as we calibrate the value of $\overline{t}^H$ to ensure that the overall consumer price of housing services remains equal to its initial value, $\overline{P}^H$, so that (1.13), (1.17) and (1.18) imply:

$$p_H = p_h = \left(1 + \overline{t}^H\right)\left(r + \delta\right) = \overline{P}^H. \tag{5.24}$$

In a similar way we may assume that all other consumer goods are uniformly taxed at the tax-exclusive rate $\overline{t}$ which ensures that the consumer price index for other goods remains equal to its pre-reform value $\overline{P}_o$:

$$\overline{P}_o = 1 + \overline{t}. \tag{5.25}$$

Note that a housing tax regime equivalent to the uniform housing tax $\overline{t}^H$ could be implemented through a property tax regime where rental housing is subject to the property tax rate $\hat{\tau}$ and owner-occupied housing is taxed at the rate $\hat{\tau} + t^r r$, provided the rental property tax rate is chosen to satisfy the condition $(1+t^H)(r+\hat{\tau}+\delta) = (1+\overline{t}^H)(r+\delta) = \overline{P}^H$.

We now consider a reform involving a switch to uniform taxation of housing and other goods at a common tax-exclusive rate t which keeps the overall consumer price index (and hence consumer welfare) constant. From (1.8), (5.24) and (5.25) it follows that this reform must satisfy

$$dP = 0 \quad \Rightarrow \quad \mu \overline{P}_H^{-\sigma} \cdot dP_H + (1-\mu)\overline{P}_o^{-\sigma} \cdot dP_o = 0 \quad \Rightarrow$$
$$\mu \overline{P}_H^{-\sigma}(r+\delta) \cdot dt^H + (1-\mu)\overline{P}_o^{-\sigma} \cdot dt = 0. \qquad (5.26)$$

Given the assumption that the new housing tax rate t^H is equal to t after the reform, we have

$$dt \equiv t - \overline{t} \quad \text{and} \quad dt^H \equiv t^H - \overline{t}^H = t - \overline{t}^H. \qquad (5.27)$$

Inserting (5.24), (5.25) and (5.27) in (5.26) and rearranging, we get:

$$t = \omega_o \overline{t} + (1-\omega_o)\overline{t}^H, \qquad \omega_o \equiv \frac{1}{1+Z}, \qquad Z \equiv (r+\delta)^{1-\sigma}\left(\frac{\mu}{1-\mu}\right)\left(\frac{1+\overline{t}}{1+\overline{t}^H}\right)^{\sigma}. \qquad (5.28)$$

The change in revenue induced by the reform is

$$dR = \overbrace{dt^H \cdot (r+\delta)C_H + dt \cdot C_o}^{\text{static revenue effect}} + \overbrace{\overline{t}^H (r+\delta)\frac{dC_H}{dP_H}\frac{dP_H}{dt^H} \cdot dt^H + \overline{t}\frac{dC_o}{dP_o}\frac{dP_o}{dt} \cdot dt}^{\text{dynamic revenue effect}}. \qquad (5.29)$$

By inserting (1.6) and (1.7) in (5.29) and using (5.26), one can easily show that the static revenue effect is once again zero. The same equations plus (5.24), (5.25) and (5.28) can be shown to imply that the dynamic (and thus the total) revenue effect of the move towards uniform taxation of all consumption is

$$\frac{dR}{PC} = \sigma(1-\mu)(1-\omega_o)\left(\frac{\overline{P}_o}{P}\right)^{1-\sigma}\left[\left(\frac{\overline{t}^H}{1+\overline{t}^H}\right)-\left(\frac{\overline{t}}{1+\overline{t}}\right)\right]\left(\frac{\overline{t}^H-\overline{t}}{1+\overline{t}}\right)>0, \quad (5.30)$$

where the positive sign of the revenue effect follows from the fact that μ and ω_o always take values between zero and unity.

The analysis leading to (5.30) ignores non-fiscal externalities. In the presence of consumption externalities and Pigovian excise taxation, the numerator in the term $\overline{t}/(1+\overline{t})$ in the square bracket in (5.30) must be adjusted for the VAT-inclusive excise tax component to account for the fact that a change in the consumption of non-housing goods is associated with a change in external costs equal to the change in excise tax revenue. The adjusted value of the numerator can be calculated from (3.4) and (3.8) by setting $t^n = t_v^n$.

6 Empirical application: data needs

Based on the formulas derived in the previous sections, this section provides a summary of the data needed to calculate marginal deadweight losses from the main tax instruments as well as the deadweight losses from non-uniform taxation. The notation in the following is the same as that used in the previous sections.

6.1 Income data

$WL =$ aggregate wage bill (including employers' social security contributions)

$(\rho - \delta_k)K =$ aggregate net business profits earned on domestic business investment (gross profit minus economic depreciation)

$rPS =$ aggregate income from wealth earned by domestic residents (including imputed returns to owner-occupied housing and returns to savings channelled through pension funds and life insurance companies etc.)

$$b_1 \equiv \frac{B_1}{WL} = \frac{\text{public after-tax transfers to people of working age}}{\text{aggregate wage bill}}$$

$$+ \overbrace{\text{marginal labour income tax rate}}^{t^w} \text{ - average labour income tax rate}$$

$$b_2 \equiv \frac{B_2}{WL} = \frac{\text{public after-tax transfers to people above working age}}{\text{aggregate wage bill}}$$

$\rho K =$ total business profits before depreciation and interest

$\rho_c K_c =$ profits before depreciation and interest in closely held firms (unincorporated firms plus closely held corporations)

$\rho_{cc} K_{cc} =$ profits before interest and depreciation in closely held corporations

$\rho_{cp} K_{cp} =$ profits before interest and depreciation in unincorporated firms

$(r+\pi) S^I =$ nominal return to financial wealth held by institutional investors (pension funds and life insurance companies etc.)

$(r+\pi) S^F =$ nominal return to financial wealth held directly by households (wealth not held through institutional investors)

6.2 Data on consumption

$PC =$ total private consumption
$P_H C_H =$ total consumption of housing services
$p_H C_H =$ consumption of owner-occupied housing services
$p_h C_h =$ consumption of rental housing services
$p_n C_n =$ consumption of non-housing goods in VAT category n

6.3 Tax rates

$t_v^n =$ VAT rate on consumption in VAT category n
$t_e^n =$ average excise tax rate on goods in VAT category n
$t^H =$ VAT rate on sale of newly constructed housing units
$\tau^H =$ property tax rate on owner-occupied housing, measured relative to the current market value of the property
$\tau^h =$ property tax rate/subsidy rate on rental housing, measured relative to the current market value of the property
$t^w =$ average value of the marginal direct tax rate on gross labour income (including social security taxes paid by employers and employees)

t^I = marginal effective tax rate on return to savings channelled through institutional investors
t^F = marginal personal tax rate on capital income

To calculate the variable b_1 specified in section 6.1, one also needs an estimate of the average value of the average direct tax rate on labour income across the working population. Further, to estimate the excise tax rate t_e^n appearing in formula (5.11), one needs data on the total revenue from excises levied on consumption of each goods category n. These revenue figures may then be divided by the tax-exclusive value of total consumption in each goods category to obtain an estimate of t_e^n .

In addition to information on the above tax rates, the estimation of marginal effective tax rates on investment requires information on a number of parameters in the system of business income taxation. These parameters are specified in the appendix.

6.4 Elasticities

The application of the formulas for effective tax wedges and for deadweight loss requires estimates of (or assumptions on) the following elasticities:

Elasticities of substitution in consumption and portfolio choice
σ = elasticity of substitution between housing services and other goods
σ_o = elasticity of substitution between the various 'other goods'
σ_h = elasticity of substitution between owner-occupied housing and rental housing
ϕ = elasticity of substitution between institutional saving and 'free' financial saving

Elasticities of substitution in production
σ_k = elasticity of substitution between capital invested in widely and closely held firms
σ_c = elasticity of substitution between capital invested in closely held corporations and proprietorships

Elasticities of factor supply and factor demand
ε_{ρ}^{K} = elasticity of aggregate capital demand w.r.t. the user cost of capital
ε_{w}^{L} = compensated elasticity of labour supply w.r.t. the real marginal after-tax wage rate
ε_{r}^{L} = compensated elasticity of labour supply w.r.t. the real after-tax interest rate
ε_{w}^{S} = compensated elasticity of aggregate saving w.r.t. the real marginal after-tax wage rate
ε_{r}^{S} = compensated elasticity of aggregate saving w.r.t. the real after-tax interest rate

6.5 Depreciation rates

δ = real rate of economic depreciation of housing capital
δ_k = average real rate of economic depreciation of business capital

6.6 Estimating consumption weights

Given the data on consumption listed in section 6.2 and the substitution elasticities in consumption listed in section 6.4, one can estimate the weight parameters μ, η, and β_n entering the formulas for the various price indices. For example, equations (1.11) through (1.13) imply that

$$\eta = s^H \cdot p_H^{\sigma_h - 1} \cdot V, \qquad 1-\eta = \left(1-s^H\right) \cdot p_h^{\sigma_h - 1} \cdot V,$$
$$s^H \equiv p_H H / P_H C_H, \qquad V \equiv \eta p_H^{1-\sigma_h} + \left(1-\eta\right) p_h^{1-\sigma_h}. \tag{6.1}$$

Adding the first two equations in (6.1) and solving for V, we obtain

$$V = \frac{1}{s^H p_H^{\sigma_h - 1} + \left(1-s^H\right) p_h^{\sigma_h - 1}}. \tag{6.2}$$

The variables p_H and p_h are given by (1.17) and (1.18) which can be estimated from the data on effective tax rates and depreciation rates. This in turn allows an estimation of η on the basis of (6.2) and (6.1).

In a similar way, by using (1.14), (1.15) and the constraint $\sum_n \beta_n = 1$, one can show that

$$\beta_n = s_n \cdot \left(1+t^n\right)^{\sigma_o - 1} \cdot X, \qquad s_n \equiv \frac{p_n x_n}{P_o C_o}, \qquad X \equiv \frac{1}{\sum_n s_n \left(1+t^n\right)^{\sigma_o - 1}}, \tag{6.3}$$

while (1.6) through (1.8) imply

$$\mu = s_H P_H^{\sigma-1} Z, \qquad s_H \equiv \frac{P_H C_H}{PC}, \qquad Z \equiv \frac{1}{s_H P_H^{\sigma-1} + \left(1-s_H\right) P_o^{\sigma-1}}. \tag{6.4}$$

Once η and β_n have been estimated from (6.1) through (6.3), once can calculate the price indices P_H and P_o from (1.13) and (1.15), allowing μ to be derived from (6.4).

6.7 Estimating portfolio weights

To estimate the deadweight loss from non-neutral tax treatment of different forms of financial saving, one needs an estimate of the weight parameter γ in (1.28). This may be obtained by rearranging (1.30) and (1.31) and using (1.29) and (1.32) to find

$$\gamma = s^I \left(1-t^I\right)^{-(\phi+1)} \hat{\theta}, \qquad 1-\gamma = \left(1-s^I\right)\left(1-t^F\right)^{-(\phi+1)} \hat{\theta},$$

$$s^I \equiv \frac{1}{1+\left(\frac{1-t^F}{1-t^I}\right)\left(\frac{S^F}{S^I}\right)}, \qquad \hat{\theta} \equiv \gamma\left(1-t^I\right)^{\phi+1} + (1-\gamma)\left(1-t^F\right)^{\phi+1}. \tag{6.5}$$

Adding the two first equations in (6.5) and solving for $\hat{\theta}$, one gets

$$\hat{\theta} = \frac{1}{s^I \left(1-t^I\right)^{-(\phi+1)} + \left(1-s^I\right)\left(1-t^F\right)^{-(\phi+1)}}. \tag{6.6}$$

From (6.5) one may then derive γ, given the observed ratio s^I and the estimated/assumed value of the elasticity ϕ . Note that the ratio S^F / S^I equals the ratio $(r+\pi)S^F / (r+\pi)S^I$ for which data are assumed to be available. If t_s^I and t_s^F are the statutory rates of tax on the *nominal* return to the two forms of saving, the implied tax rates on the real rates of return are given as

$$t^I \equiv \frac{t_s^I\left(r+\pi\right)}{r}, \qquad t^F \equiv \frac{t_s^F\left(r+\pi\right)}{r}. \tag{6.7}$$

From these relationships plus the estimate of $(r+\pi)S^I / (r+\pi)S^F = S^I / S^F$ one can calculate the value of the fraction s^I appearing in (6.5) and (6.6).

6.8 Estimating sectoral weights

Finally, one needs estimates of the weights ϖ and χ in (2.19) and (2.20) to calculate the efficiency loss from non-neutral tax treatment of different forms of business organisation. From (2.22), (2.26) through (2.28) one finds that

$$\chi = s_c \rho_{cc}^{\sigma_c - 1} \hat{\rho}_c, \qquad 1-\chi = \left(1-s_c\right) \rho_{cp}^{\sigma_c - 1} \hat{\rho}_c,$$
$$s_c \equiv \frac{\rho_{cc} K_{cc}}{\rho_c K_c}, \qquad \hat{\rho}_c \equiv \frac{1}{\chi \rho_{cc}^{1-\sigma_c} + \left(1-\chi\right)\rho_{cp}^{1-\sigma_c}}. \tag{6.7}$$

By adding the two first equations and solving for $\hat{\rho}_c$, one gets

$$\hat{\rho}_c \equiv \frac{1}{s_c \rho_{cc}^{\sigma_c - 1} + \left(1-s_c\right)\rho_{cp}^{\sigma_c - 1}}. \tag{6.8}$$

Equations (6.7) and (6.8) allow an estimation of χ, given estimates of ρ_{cc} and ρ_{cp} and the observed value of s_c plus the estimated/assumed value of the substitution elasticity σ_c.
Following a similar procedure, one finds from (2.21) plus (2.23) through (2.25) that

$$\varpi = s_w \rho_w^{\sigma_k - 1} \hat{\rho}, \qquad s_w \equiv \frac{\rho_w K_w}{\rho K}, \qquad \hat{\rho} \equiv \frac{1}{s_w \rho_w^{\sigma_k - 1} + \left(1-s_w\right)\rho_c^{\sigma_k - 1}}, \tag{6.9}$$

where ρ_w and ρ_c can be calculated from (2.28) through (2.30).

Appendix A
Estimation of marginal effective tax wedges on business income: A case study of Sweden

This appendix explains how to derive the cost of capital and hence the investment tax wedge for different forms of business organization. The approach to the estimation of effective tax wedges adopted here follows the tradition established by King and Fullerton (1984), discussed in detail by Sørensen (2004). For closely held companies and proprietorships the investment tax wedge will be influenced by domestic personal taxes as well as any special tax rules applying to small and/or closely held firms. These rules vary significantly from one country to another, making it difficult to specify the investment tax wedge for closely held firms in general terms. To illustrate how the King-Fullerton methodology may be adapted to fit the tax rules of a particular country, this appendix will include a case study of the current (2010) Swedish tax rules for closely held firms.

Throughout the appendix we shall use the following

Notation

a = present value of capital allowances generated by an additional unit of investment in excess of the present value of allowances for true economic depreciation
i^n = imputed nominal rate of return on business equity
r = real rate of interest before tax
r^a = real after-tax rate of return required by investors
δ_k = real exponential rate of economic depreciation

ρ = real user cost of capital (including depreciation)
π = rate of inflation
τ = statutory corporate income tax rate
τ^c = ordinary personal capital income tax rate
τ^d = personal tax rate on dividends
τ^g = effective personal tax rate on accrued capital gains on shares
τ^w = effective marginal personal tax rate on labour income (including social security tax)
s = social security tax rate
PV = present value of pre-tax cash flows generated by an additional unit of investment
PVT = present value of additional tax payments generated by an extra unit of investment

The marginal investment project and the cost of capital

We wish to derive the required real pre-tax rate of return on an investment project that is just barely worth undertaking. Such a marginal investment project has a zero net present value and therefore satisfies the break-even condition:

$$PV - PVT = 0. \qquad \text{(A.1)}$$

Suppose that in each period the firm retains and reinvests an amount of gross profit equal to the true economic depreciation of its existing assets so as to maintain the real value of its capital stock. In the notation above, the nominal amount that needs to be reinvested in period *t* will then be $\delta_k e^{\pi t}$. For a firm financing its initial investment by equity and applying the real discount rate *r*, the present value of the pre-tax nominal cash flows from an additional unit of investment is thus given by

$$PV = \int_0^\infty (\rho - \delta_k) e^{[\pi-(r+\pi)]t} dt - 1 = \int_0^\infty (\rho - \delta_k) e^{-rt} dt - 1 = \frac{\rho - \delta_k}{r} - 1, \qquad \text{(A.2)}$$

since the initial investment outlay is 1 and the subsequent annual real pre-tax cash flow gross of depreciation is equal to ρ. The magnitude $\rho - \delta_k$ in (A.2) is usually referred to as the cost of capital, i.e. the required minimum real rate of return on investment, measured before tax but after deduction for true economic depreciation. The investment tax wedge is obtained by deducting the international real interest rate *r* from the cost of capital.

Alternatively, the investment may be financed by debt. In that case we assume that the nominal stock of debt is allowed to rise over time in line with the rate of inflation so that the initial debt-asset ratio is kept constant (recall that since the real capital stock is maintained over time through reinvestment corresponding to true economic depreciation, the nominal capital stock will grow at the rate of inflation). With this assumption the positive cash flow from additional nominal borrowing in period *t* is $\pi e^{\pi t}$, and the nominal amount of interest payments in period *t* is $(r+\pi)e^{\pi t}$. The present value of the pre-tax nominal cash flows from an additional unit of debt-financed investment is therefore equal to

$$PV = \int_0^\infty \left[\rho - \delta_k + \pi - (r+\pi)\right] e^{[\pi-(r+\pi)]t} dt = \int_0^\infty (\rho - \delta_k - r) e^{-rt} dt = \frac{\rho - \delta_k - r}{r}, \quad \text{(A.3)}$$

which is seen to be identical to (A.2).

The formulas (A.2) and (A.3) are relevant for all organizational forms. However, when applying these formulas to closely held firms, one must replace the pre-tax real interest rate *r* by the after-tax real interest rate r^d, since the owners of such firms are subject to domestic personal taxes and therefore use an after-tax discount rate.

To derive the cost of capital from formula (1), one also needs to derive the present value of the tax payments generated by the marginal investment. This magnitude (*PVT*) will depend on the particular tax rules applied to the various forms of business organization, as explained in the sections below.

The cost of capital for widely held corporations

Widely held company: finance by equity

When calculating *PVT*, it is useful to split total capital allowances for tax purposes into a component corresponding to the true economic depreciation and a residual component capturing accelerated depreciation for tax purposes plus any other special allowances such as investment tax credits and other tax-related investment subsidies that might be available to firms. Recalling that the present value of such accelerated depreciation etc. per unit of investment is denoted by a, the present value of the tax payments generated by an equity-financed investment (denoted by superscript e) undertaken by a widely held company (indicated by subscript w) thus becomes:

$$PVT_w^e = \int_0^\infty \tau\left(\rho_w^e - \delta_k\right)e^{-rt}dt - \tau a = \tau\left(\frac{\rho_w^e - \delta_k}{r} - a\right). \quad \text{(A.4)}$$

Note that as a result of inflation, the nominal value of the firm's asset stock rises by the amount $\pi e^{\pi t}$ in period t. However, since the firm is assumed to hold on to its investment, this accrued nominal capital gain is not realized. Equation (A.4) therefore assumes that the nominal capital gain on the firm's asset stock goes untaxed, as is usually the case in practice, as long as the gain is not realized.

Inserting (A.4) and (A.2) into (A.1) and solving for the cost of capital, we get:

$$\rho_w^e - \delta_k = \left(\frac{r}{1-\tau}\right)\left(1 - \tau a\right). \quad \text{(A.5)}$$

Widely held company: finance by debt

In the case of debt finance (indicated by a d-superscript), the company gets a nominal interest deduction amounting to $\left(r+\pi\right)e^{\pi t}$ in period t, so the present value of tax payments becomes:

$$PVT_w^d = \int_0^\infty \tau\left[\rho_w^d - \delta_k - \left(r+\pi\right)\right]e^{\left[\pi-(r+\pi)\right]t}dt - \tau a = \tau\left[\frac{\rho_w^d - \delta_k - \left(r+\pi\right)}{r} - a\right]. \quad \text{(A.6)}$$

Substituting (A.6) and (A.3) into (A.1) and rearranging, we find the cost of capital under debt finance:

$$\rho_w^d - \delta_k = r - \left(\frac{\tau}{1-\tau}\right)\left(\pi + ra\right). \quad \text{(A.7)}$$

The cost of capital for sole proprietorships in Sweden[8]

In many countries small closely held firms are subject to special tax rules. When applying the King-Fullerton methodology to derive the cost of capital for these firms, the formula for the present value of tax payments therefore has to be modified accordingly.

To illustrate, this section derives the cost of capital for sole proprietorships subject to the Swedish dual income tax regime. Under this regime the total business income of proprietors is split into a capital income component, calculated as an imputed rate of return on the net equity of the business, and a residual income component which is taxed as labour income in so far as the income is distributed to the owner. If the residual income is retained in the firm and allocated to a so-called expansion fund, it is subject to a preliminary tax at a rate equal to the corporate income tax rate. When previously retained income is subsequently distributed, it is 'grossed up' by the amount of tax already paid, and the resulting pre-tax income is then taxed as labour income, with a credit for the prepaid tax being granted against the final labour income tax bill (see Sørensen (2008, Ch. 3) for a detailed description of these rules). The subsections below illustrate how the present value of tax payments may be estimated under this tax regime.

Sole proprietor: finance by new equity

Consider a sole proprietor who injects one krona of *new* equity into his business, thereby increasing his imputed nominal capital income by the amount i^n in every future year. Assuming that the actual return on the investment exceeds the imputed return, the marginal income from the project will be taxed at the effective marginal labour income tax rate τ^w, but at the same time the proprietor will save an amount of tax equal to the difference $\tau^w - \tau^c$ between the marginal effective labour income tax rate and the capital income tax rate (τ^c) on that part of the return from the project which is taxed as capital income. Recalling that a proprietor subject to personal income tax will discount future nominal cash flows with the nominal after-tax discount rate $r^a + \pi$, the present value of the future tax bill generated by the marginal investment thus becomes

[8] This main section and the following one draws heavily on Sørensen (2008, Appendix 4.2).

$$PVT_{cp}^{ne} = \int_0^\infty \tau^w \left(\rho_{cp}^{ne} - \delta_k\right) e^{\left[\pi - \left(r^a + \pi\right)\right]t} dt - \int_0^\infty \left(\tau^w - \tau^c\right) i^n e^{-\left(r^a + \pi\right)t} dt - \tau^w a$$
$$= \tau^w \left(\frac{\rho_{cp}^{ne} - \delta_k}{r^a}\right) - \left(\frac{i^n}{r^a + \pi}\right)\left(\tau^w - \tau^c\right) - \tau^w a, \tag{A.8}$$

where the superscript *ne* indicates that investment is financed by new equity, and the subscript *cp* refers to a closely held firm organized as a proprietorship. Note that since the magnitude $\rho_{cp}^{ne} - \delta_k$ measures the *real* profit stream from the project, it is discounted at the proprietor's real discount rate r^a. By contrast, the imputed return i^n is a fixed nominal amount which does not grow in line with inflation; for this reason it is discounted at the proprietor's nominal discount rate $r^a + \pi$.

To find the cost of capital, we insert (A.8) and (A.2) into (A.1) and solve for $\rho_{cp}^{ne} - \delta_k$ to obtain

$$\rho_{cp}^{ne} - \delta_k = \frac{r^a}{1-\tau^c} + \left[\frac{r^a}{1-\tau^c} - \left(\frac{r^a}{r^a + \pi}\right) i^n\right]\left(\frac{\tau^w - \tau^c}{1-\tau^w}\right) - \left(\frac{\tau^w r^a a}{1-\tau^w}\right). \tag{A.9}$$

This formula is identical to equation (19) in Lindhe, Södersten and Öberg (2003, p. 13) in the case considered by those authors where the inflation rate is (implicitly) assumed to be zero ($\pi = 0$) and depreciation for tax purposes is assumed to equal the true economic depreciation ($a = 0$).

Sole proprietor: finance by retained earnings

Instead of injecting new equity, the proprietor may choose to finance the investment through *retained earnings*, making use of the expansion fund system. Since earnings retained in the expansion fund are taxed at the corporate tax rate τ, the proprietor must retain a pre-tax income of $\frac{1}{1-\tau}$ kronor to fund the 1 krona investment. By retaining this amount in the business rather than distributing it and having it taxed as labour income (assuming that total business income exceeds the imputed return to equity), the proprietor saves an amount of labour income tax equal to $\frac{\tau^w}{1-\tau}$, but at the same time he must pay an amount of tax equal to $\frac{\tau}{1-\tau}$ on the profit retained. Further, recall that the effect of accelerated depreciation is modelled as if the firm is granted an immediate capital allowance *a* per krona of investment in addition to the future deductions for true economic depreciation. During the year of investment, the decision to retain an additional amount of profit thus implies the following

$$\text{Tax saving at the time of investment:} \quad \frac{\tau^w - \tau}{1-\tau} - \tau^w a. \quad \text{(A.10)}$$

Since profits retained in the expansion fund do not add to the equity base for the calculation of the proprietor's imputed capital income, all of the future income from the project will be taxed as labour income, generating a tax bill with the following present value:

$$\text{Present value of future taxes:} \quad \int_0^\infty \tau^w \left(\rho_{cp}^{re} - \delta_k\right) e^{-r^a t} dt = \left(\frac{\rho_{cp}^{re} - \delta_k}{r^a}\right) \tau^w, \quad \text{(A.11)}$$

where the superscript *re* indicates that investment is financed by retained earnings. Combining (A.10) and (A.11), we get the net present value of the additional tax liability implied by the project:

$$PVT_{cp}^{re} = \left(\frac{\rho_{cp}^{ne} - \delta_k}{r^a}\right) \tau^w - \left(\frac{\tau^w - \tau}{1-\tau}\right) - \tau^w a. \quad \text{(A.12)}$$

The cost of capital may now be found by inserting (4.12) and (4.2) into (4.1) and solving for $\rho_{cp}^{ne} - \delta_k$:

$$\rho_{cp}^{ne} - \delta_k = \frac{r^a}{1-\tau} - \left(\frac{\tau^w r^a a}{1-\tau^w}\right). \quad \text{(A.13)}$$

Equation (4.13) is identical to formula (21) in Lindhe et alia (2003, p. 14) in the case considered by these authors where there is no accelerated depreciation ($a = 0$).

Sole proprietor: finance by debt

As another alternative, the proprietor may finance investment by *debt* to benefit from the deductibility of interest. Assuming that the proprietor's investment return net of interest is paid out and taxed as labour income, and recalling that the nominal stock of debt is allowed to grow in line with the growth in the nominal value of the asset caused by inflation, the present value of the total tax liability triggered by the marginal investment project will be

$$PVT_{cp}^{d} = \int_{0}^{\infty} \tau^{w} \left[\rho_{cp}^{d} - \delta_{k} - \left(r + \pi \right) \right] e^{\left[\pi - \left(r^{a} + \pi \right) \right] t} dt - \tau^{w} a$$

$$= \tau^{w} \left[\frac{\rho_{cp}^{d} - \delta_{k} - \left(r + \pi \right)}{r^{a}} \right] - \tau^{w} a, \qquad \text{(A.14)}$$

where the superscript d stands for 'debt finance'.

Inserting (A.14) and (4.2) into (4.1) and solving for $\rho_{cp}^{d} - \delta_{k}$, we get the proprietor's cost of capital for debt-financed investment:

$$\rho_{cp}^{d} - \delta_{k} = r - \left(\frac{\tau^{w}}{1 - \tau^{w}} \right) \left(\pi + r^{a} a \right). \qquad \text{(A.15)}$$

The cost of capital for closely held corporations in Sweden

We now consider a closely held company owned by a so-called qualified shareholder subject to the Swedish tax rules for *fåmansföretag* (companies with few owners). The purpose of these rules is to prevent shareholders working in their own company from transforming high-taxed labour income into low-taxed dividend income under the Swedish dual income tax regime. The rules for *fåmansföretag* (sometimes referred to as the 3:12 rules) are complex and are described more carefully in Sørensen (2008, Ch. 3), but the main principles are as follows:

A company is considered to have few owners if more than 50 percent of the voting shares in the company are controlled by at most four shareholders. To be deemed a qualified shareholder in a company with few owners, the shareholder must be active in the company to a significant degree so that his activity has a significant influence on the income generated by the company. When the holder of a qualified share receives a dividend from the company, the dividend is split into a capital income component and a labour income component. Dividends below the limit for the so-called normal dividend (normalutdelning) are taxed as capital income, while dividends exceeding the 'normal' level are taxed as labour income. If the limit for the normal dividend exceeds the actual dividend, the difference –referred to as the Unutilized Distribution Potential (UDP)[9] – may be carried forward with interest and utilized in a later year.

The limit for the normal dividend is calculated as the sum of three components: 1) An imputed return on the purchase price of the share, 2) The sum of all UDP amounts from previous years, carried forward with interest, and 3) An additional amount based on the company's wage bill, henceforth termed the Wage-Based Allowance (WBA). The wage bill serving as a basis for calculating the WBA includes the wage or salary paid to the shareholder himself.

As explained in Sørensen (2008, Ch. 3), the prevailing tax rates imply that the optimal distribution policy for a qualified shareholder subject to the progressive central government labour income tax is to pay himself a dividend equal to the normal dividend and withdraw any further income from the company in the form of wages.

Qualified shareholder: finance by new equity

Suppose such a qualified shareholder finances one krona of investment by injecting *new equity* into his business. Potentially his imputed normal dividend then goes up by the amount i^n (the imputed rate of return) in every future year, allowing him to transform highly taxed wage income into low-taxed dividend income. However, a qualified shareholder may include a certain fraction (denoted here by ω) of his wage in his normal dividend, so when he reduces his wage withdrawal by one krona, the normal

[9] In Swedish the UDP is sometimes referred to as "sparat utdelningsutrymme" or "sparat gränsbelopp".

dividend falls by ω kronor.[10] As a consequence, the net increase in the normal dividend (ΔD) made possible by the injection of one krona of new equity into the company becomes

$$\Delta D = i^n - \omega \cdot \Delta w = i^n - \omega \cdot \left(\frac{\Delta W}{1+s} \right), \tag{A.16}$$

where ΔW is the absolute reduction in the company's wage cost including the social security tax, and $\Delta w = \frac{\Delta W}{1+s}$ is the absolute reduction in the wage paid out to the shareholder net of the social security tax s. The profit underlying the dividend is subject to corporation tax, so when the company cuts the deductible gross wage to the shareholder by the amount ΔW, it can only increase its dividend payment by the amount $\Delta D = (1-\tau)\Delta W$. Inserting this into (A.16), it follows that the amount of pre-tax business income that may be transformed from wage income into dividend income when the shareholder's equity base increases by one krona must satisfy the constraint

$$\Delta W (1-\tau) = i^n - \omega \cdot \left(\frac{\Delta W}{1+s} \right) \quad \Leftrightarrow \quad \Delta W = \frac{i^n}{1-\tau+\hat{\omega}}, \qquad \hat{\omega} \equiv \frac{\omega}{1+s}, \tag{A.17}$$

so the net increase in the normal dividend becomes

$$\Delta D = (1-\tau)\Delta W = i^n \left(\frac{1-\tau}{1-\tau+\hat{\omega}} \right). \tag{A.18}$$

From (A.17) and (A.18) it follows that the transformation of wage income into dividend income will lead to the following annual nominal reduction in personal tax and social security tax (which is included in the marginal effective labour income tax rate (τ^w)):

$$\tau^w \Delta W - \tau^d \Delta D = \left(\frac{i^n}{1-\tau+\hat{\omega}} \right) \left[\tau^w - \tau^d (1-\tau) \right]. \tag{A.19}$$

[10] Since time is treated as a continuous variable for simplicity, we assume that a fall in wage payments has an immediate impact on the normal dividend, whereas in practice the normal dividend for the current year depends on wage payments during the previous year. The inaccuracy implied by this simplification is likely to be minor.

At the same time, since wages are deductible from the corporate income tax base, the result in (A.17) implies that the cut in the shareholder's wage income will generate the following annual nominal increase in the corporate tax bill:

$$\tau \Delta W = \frac{\tau i^n}{1-\tau+\hat{\omega}}. \tag{A.20}$$

Combining (A.19) and (A.20), we obtain the

Annual nominal tax saving resulting from the increase in the equity base:

$$\tau^w \Delta W - \tau \Delta W - \tau^d \Delta D = \left(\frac{i^n}{1-\tau+\hat{\omega}}\right)\left[\tau^w - \tau - \tau^d (1-\tau)\right]. \tag{A.21}$$

Note that since the basis value of the shares in the company is not indexed to inflation, the imputed rate of return i^n is a fixed nominal amount, so the future annual tax savings recorded in (A.21) should be discounted at the shareholder's nominal discount rate.

Equation (A.21) does not include the effect of the wage-based allowance generated by wages paid to the company's employees. To account for this effect, we allow for the possibility that when the firm's capital stock is increased by one krona, the total *real* annual wage bill paid to the employees may go up by some amount A, reflecting the possible need for increased manpower to operate the larger capital stock. [11]Hence one krona of investment will ceteris paribus increase the real annual normal dividend by the amount ωA through an increase in the wage-based allowance. Following a procedure identical to the one described above, we can therefore derive the following

[11] On average the parameter A will be positive, given that firms always use some combination of labour and capital in production. However, at the margin of investment A will be negative if labour and capital are substitutes in the production process, and positive if labour and capital are complementary factors of production, as explained in Sørensen (2008, Ch. 4).

Annual real tax saving due to higher allowance for wages paid to employees:

$$\left(\frac{\hat{\omega}A}{1-\tau+\hat{\omega}}\right)\left[\tau^{w}-\tau-\tau^{d}\left(1-\tau\right)\right] \tag{A.22}$$

Since *A is a real* amount, the tax saving in (A.22) must be discounted at the shareholder's real discount rate.

The wage-based allowance also affects the net tax rate on the distributed investment return. As the yield from the investment generates higher wage payments to the shareholder, the wage-based allowance also goes up, enabling the shareholder to distribute part of the investment yield as a higher normal dividend. In particular, since $\Delta D=\hat{\omega}\Delta W$, the sum of the higher gross (real) wages and (real) dividends generated by the distribution of the real pre-tax return $\rho_{cc}^{ne}-\delta_{k}$ (where *cc* stands for 'closely held corporation') is given by the constraint

$$\Delta W+\frac{\Delta D}{1-\tau}=\rho_{cc}^{ne}-\delta_{k} \quad \Rightarrow \quad \Delta W+\frac{\hat{\omega}\Delta W}{1-\tau}=\rho_{cc}^{ne}-\delta_{k},$$

implying that

$$\Delta W=\left(\frac{1-\tau}{1-\tau+\hat{\omega}}\right)\left(\rho_{cc}^{ne}-\delta_{k}\right) \quad \text{and} \quad \Delta D=\left(\frac{1-\tau}{1-\tau+\hat{\omega}}\right)\hat{\omega}\left(\rho_{cc}^{ne}-\delta_{k}\right).$$

These changes in wages and dividends generate the following (real) annual tax liabilities:

Increase in wage tax: $\tau^{w}\Delta W=\tau^{w}\left(\frac{1-\tau}{1-\tau+\hat{\omega}}\right)\left(\rho_{cc}^{ne}-\delta_{k}\right)$

Increase in dividend tax: $\tau^{d}\Delta D=\tau^{d}\left(\frac{1-\tau}{1-\tau+\hat{\omega}}\right)\hat{\omega}\left(\rho_{cc}^{ne}-\delta_{k}\right)$

Increase in corporate income tax: $\frac{\tau\Delta D}{1-\tau}=\tau\left(\frac{\hat{\omega}}{1-\tau+\hat{\omega}}\right)\left(\rho_{cc}^{ne}-\delta_{k}\right)$

Note that the increase in the corporate income tax liability arises because an amount $\Delta D/(1-\tau)$ of the pre-tax return is distributed as a dividend which is not deductible from the corporate income tax base. Adding up the above changes in tax payments, we get the

Annual real tax increase generated by the distribution of the investment return:

$$\tau^w \Delta W + \tau^d \Delta D + \frac{\tau \Delta D}{1-\tau} = \tau^a \left(\rho_{cc}^{ne} - \delta_k\right),$$

$$\tau^a \equiv \left(\frac{1-\tau}{1-\tau+\hat{\omega}}\right)\tau^w + \left(\frac{\hat{\omega}}{1-\tau+\hat{\omega}}\right)\left[\tau + \tau^d (1-\tau)\right], \tag{A.23}$$

where the average tax rate on the distributed investment return, τ^a, is seen to be a weighted average of the tax rate on labour income and the total corporate and personal tax rate on dividends, $\tau + \tau^d (1-\tau)$. Again, since $\rho_{cc}^{ne} - \delta_k$ is a real rate of return, the stream of tax payments specified in (A.23) should be discounted at the shareholder's real discount rate r^a.

Using (A.21), (A.22) and (A.23), it follows that the present value of the future tax bill generated by the additional investment becomes:

$$PVT_{cc}^{ne} = \int_0^\infty \left\{\tau^a \left(\rho_{cc}^{ne} - \delta_k\right) - \left(\tfrac{\hat{\omega}A}{1-\tau+\hat{\omega}}\right)\left[\tau^w - \tau - \tau^d (1-\tau)\right]\right\} \cdot e^{-r^a t} dt$$
$$- \int_0^\infty \left(\tfrac{i^n}{1-\tau+\hat{\omega}}\right)\left[\tau^w - \tau - \tau^d (1-\tau)\right] \cdot e^{-\left(r^a+\pi\right)t} dt - \tau^a a \quad \Leftrightarrow$$

$$PVT_{cc}^{ne} = \tau^a \left(\frac{\rho_{cc}^{ne} - \delta_k}{r^a}\right) - \left(\frac{\hat{\omega}A}{r^a} + \frac{i^n}{r^a+\pi}\right)\left[\frac{\tau^w - \tau - \tau^d (1-\tau)}{1-\tau+\hat{\omega}}\right] - \tau^a a. \tag{A.24}$$

To obtain the cost of capital, we insert (4.2) and (4.24) into (4.1) and solve for $\rho_{cc}^{ne} - \delta_k$:

$$\rho_{cc}^{ne} - \delta_k = \frac{r^a}{1-\tau^a} - \left[\hat{\omega} A + \left(\frac{r^a}{r^a + \pi}\right) i^n\right]\left[\frac{\tau^w - \tau - \tau^d(1-\tau)}{(1-\tau+\hat{\omega})(1-\tau^a)}\right] - \left(\frac{\tau^a r^a a}{1-\tau^a}\right). \quad \text{(A.25)}$$

This formula is identical to equation (8) in Lindhe et alia (2003, p. 9) in the case considered by those authors where $\hat{\omega} = \pi = 0$, again confirming that the present framework for calculating effective tax rates is just a generalisation of that developed by previous authors.[12]

Qualified shareholder: finance by retained earnings

Consider next the alternative case where the shareholder chooses to finance the investment by retained earnings, that is, by foregoing some wage and dividend income in the year of investment.

Ideally the shareholder would like to finance all of the investment through a reduction in his high-taxed wage income ($\frac{\Delta W}{1+s}$), but since a lower wage reduces the normal dividend via a smaller wage-based allowance, he will have to finance part of the investment through a drop in his low-taxed dividend income (ΔD). The fall in the company's gross wage bill increases the corporate tax bill by $\tau\Delta W$ while reducing the wage-based allowance (and hence the normal dividend) by the amount $\omega\frac{\Delta W}{1+s} = \hat{\omega}\Delta W$, so the total drop in the shareholder's pre-tax wage and dividend income needed to finance an extra krona of investment is determined by the constraint

$$\Delta W + \Delta D = 1 + \tau\Delta W \quad \Rightarrow \quad \Delta W + \hat{\omega}\Delta W = 1 + \tau\Delta W \quad \Rightarrow$$

$$\Delta W = \frac{1}{1-\tau+\hat{\omega}} \qquad \Delta D = \frac{\hat{\omega}}{1-\tau+\hat{\omega}} \qquad \text{(A.26)}$$

From (A.26) we obtain the

Reduction in wage tax and dividend tax in year of investment:

[12] To reproduce the formula derived by Lindhe et alia (2003), one must use (A.23) which implies that $\tau^a = \tau^w$ for $\hat{\omega} = 0$.

$$\tau^w \Delta W + \tau^d \Delta D = \frac{\tau^w + \hat{\omega}\tau^d}{1-\tau+\hat{\omega}} \quad \text{(A.27)}$$

After-tax shareholder income foregone in year of investment:

$$\left(1-\tau^w\right)\Delta W + \left(1-\tau^d\right)\Delta D = \frac{1-\tau^w + \hat{\omega}\left(1-\tau^d\right)}{1-\tau+\hat{\omega}} \quad \text{(A.28)}$$

To be willing to sacrifice the income stated in (A.28), the shareholder must be compensated by an after-tax capital gain which is at least as large as the net wage and dividend income foregone. For a marginal investment which is just barely worth undertaking, the pre-tax capital gain (q) on the shareholder's shares must therefore satisfy the condition

$$q\left(1-\tau^g\right) = \left(1-\tau^w\right)\Delta W + \left(1-\tau^d\right)\Delta D = \frac{1-\tau^w + \hat{\omega}\left(1-\tau^d\right)}{1-\tau+\hat{\omega}} \quad \Rightarrow$$

$$q = \frac{1-\tau^w + \hat{\omega}\left(1-\tau^d\right)}{\left(1-\tau+\hat{\omega}\right)\left(1-\tau^g\right)} \quad \text{(A.29)}$$

where τ^g is the effective personal tax rate on accrued capital gains on shares. In present value terms, the capital gain in (A.29) will trigger the following

Increase in personal capital gains tax liability in year of investment:

$$\tau^g q = \tau^g \left[\frac{1-\tau^w + \hat{\omega}\left(1-\tau^d\right)}{\left(1-\tau+\hat{\omega}\right)\left(1-\tau^g\right)}\right] \quad \text{(A.30)}$$

The reduction in the deductible wage payment to the shareholder also generates the following

Increase in corporate income tax in year of investment: $\tau\Delta W = \frac{\tau}{1-\tau+\hat{\omega}}$ (A.31)

Combining (A.27), (A.30) and (A.31), we get the

Total net increase in tax bill in year of investment: $\Delta T = \tau\Delta W + \tau^g q - \tau^w \Delta W - \tau^d \Delta D \quad \Rightarrow$

$$\Delta T = \frac{\left(1-\tau^g\right)\left(\tau-\tau^w-\hat{\omega}\tau^d\right)+\tau^g\left[1-\tau^w+\hat{\omega}\left(1-\tau^d\right)\right]}{\left(1-\tau+\hat{\omega}\right)\left(1-\tau^g\right)} \qquad \text{(A.32)}$$

As in the case of finance by new equity, the additional investment may increase the wage bill paid to the company's employees, thus triggering a higher wage-based allowance that raises the future normal dividend. The resulting annual real tax saving is still given by (A.22). Moreover, the income from the investment is still distributed and taxed at the average rate τ^a specified in (A.23). Using (A.22) and (A.32), we therefore obtain the following expression for the present value of the future tax bill generated by a qualified shareholder's investment financed by retained earnings:

$$PVT_{cc}^{re} = \Delta T + \int_0^{\infty}\left\{\tau^a\left(\rho_{cc}^{re}-\delta_k\right)-\left(\tfrac{\hat{\omega}A}{1-\tau+\hat{\omega}}\right)\left[\tau^w-\tau-\tau^d\left(1-\tau\right)\right]\right\}\cdot e^{-r^a t}dt-\tau^a a \quad \Rightarrow$$

$$PVT_{cc}^{re} = \frac{\left(1-\tau^g\right)\left(\tau-\tau^w-\hat{\omega}\tau^d\right)+\tau^g\left[1-\tau^w+\hat{\omega}\left(1-\tau^d\right)\right]}{\left(1-\tau+\hat{\omega}\right)\left(1-\tau^g\right)}$$
$$+\tau^a\left(\frac{\rho_{cc}^{re}-\delta_k}{r^a}\right)-\left(\frac{\hat{\omega}A}{r^a}\right)\left(\frac{\tau^w-\tau-\tau^d\left(1-\tau\right)}{1-\tau+\hat{\omega}}\right)-\tau^a a. \qquad \text{(A.33)}$$

By inserting (A.2) and (A.33) into (A.1) and utilizing the definition of τ^a, we can derive the cost of capital:

$$\rho_{cc}^{re}-\delta_k = \frac{r^a}{\left(1-\tau\right)\left(1-\tau^g\right)}-\frac{\hat{\omega}A\left[\tau^w-\tau-\tau^d\left(1-\tau\right)\right]}{\left(1-\tau+\hat{\omega}\right)\left(1-\tau^a\right)}-\left(\frac{\tau^a r^a a}{1-\tau^a}\right). \qquad \text{(A.34)}$$

In the absence of accelerated depreciation and the wage-based allowance ($a=\hat{\omega}=0$), this formula becomes identical to equation (11) on p. 10 in Lindhe et alia (2003).

Qualified shareholder: finance by debt

Consider finally the case where the investment is financed by *debt*. In this case too the qualified shareholder will benefit from the real annual tax reduction in (A.22) as his normal dividend will include a higher allowance for wages paid to the company's employees. Given that the investment return net of nominal interest payments is taxed at the average rate τ^a, the present value of the total tax liability triggered by the project will therefore be

$$PVT_{cc}^{d} = \int_0^\infty \tau^a \left\{ \rho_{cc}^{d} - \delta_k - (r+\pi) - \left(\tfrac{\hat{\omega}A}{1-\tau+\hat{\omega}}\right)\left[\tau^w - \tau - \tau^d (1-\tau)\right]\right\} \cdot e^{-r^a t} dt - \tau^a a \quad \Rightarrow$$

$$PVT_{cc}^{d} = \tau^a \left(\frac{\rho_{cc}^{d} - \delta_k - (r+\pi)}{r^a} \right) - \left(\frac{\hat{\omega}A}{r^a} \right)\left(\frac{\tau^w - \tau - \tau^d (1-\tau)}{1-\tau+\hat{\omega}} \right) - \tau^a a. \quad (A.35)$$

From (A.1), (A.2) and (A.35) one finds that the qualified shareholder's cost of capital for debt-financed investment is

$$\rho_{cc}^{d} - \delta_k = r - \frac{\hat{\omega}A\left[\tau^w - \tau - \tau^d (1-\tau)\right]}{(1-\tau+\hat{\omega})\left(1-\tau^a\right)} - \left(\frac{\tau^a}{1-\tau^a} \right)\left(\pi + r^a a\right). \quad (A.36)$$

When no wage-based allowance is granted ($\hat{\omega} = 0$ and $\tau^a = \tau^w$), this expression for the cost of capital is seen to be identical to the proprietor's cost of capital for debt finance (compare (A.15) to (A.36)).

Calculating the present value of accelerated depreciation allowances

Our cost-of-capital formulas include the parameter *a* which is defined as

$$a \equiv A^{tax} - A^{true}, \quad (A.37)$$

where

$$A^{true} = \int_0^\infty \delta_k e^{\left[\pi-(r+\pi)\right]t} dt = \int_0^\infty \delta_k e^{-rt} dt = \frac{\delta_k}{r} \quad (A.38)$$

is the present value of the (hypothetical) allowances for true economic depreciation, and A^{tax} is the present value of the capital allowances actually offered by the tax code.[13] The two most common methods of depreciation for tax purposes are the declining balance method, typically applied to machinery and equipment, and the straight-line method which is usually applied to buildings but sometimes also to intangible assets such as goodwill and patents. Tax systems normally do not allow the basis for depreciation to be indexed for inflation, so depreciation is based on the historical acquisition cost of the asset.

Under the declining balance method the asset is written down at a constant geometrical rate of, say, $\hat{\delta}$ per year. The present value of the depreciation allowances generated by the initial unit of investment undertaken at time zero is thus equal to

$$A_i^{tax} = \int_0^\infty \hat{\delta} e^{-\left(\hat{\delta}+r+\pi\right)t} dt = \frac{\hat{\delta}}{\hat{\delta}+r+\pi}, \tag{A.39}$$

where the subscript i refers to the initial investment. However, recall that in every subsequent year t the firm is assumed to undertake an amount of reinvestment $\delta_k e^{\pi t}$ in order to maintain the real value of its business asset. In the future year $u > t$ this amount of reinvestment will have a book value of $\delta_k e^{\pi t} \cdot e^{-\hat{\delta}(u-t)}$ in the firm's tax accounts, so the present value at time 0 of the future depreciation allowances generated by the reinvestment undertaken in year t is

$$\hat{\delta}\int_t^\infty \delta_k e^{\pi t} \cdot e^{-\hat{\delta}(u-t)} \cdot e^{-(r+\pi)u} du = \hat{\delta}\delta_k e^{\left(\pi+\hat{\delta}\right)t} \int_t^\infty e^{-\left(\hat{\delta}+r+\pi\right)u} du = \frac{\hat{\delta}\delta_k e^{-rt}}{\hat{\delta}+r+\pi}.$$

At time zero the present value of the depreciation allowances triggered by all of the firm's reinvestment undertaken after the time of the initial investment (A_r^{tax}) is thus given by

$$A_r^{tax} = \int_0^\infty \left(\frac{\hat{\delta}\delta_k}{\hat{\delta}+r+\pi} \right) e^{-rt} dt = \frac{\hat{\delta}\delta_k}{r\left(\hat{\delta}+r+\pi\right)}. \tag{A.40}$$

[13] Formula (A.38) and the subsequent formulas relate to a widely held firm using the pre-tax real discount rate r. In case the firm is closely held, one only needs to substitute the real after-tax discount rate r^a for r.

The total present value of all the depreciation allowances triggered by the firm's investment is the sum of the expressions in (A.39) and (A.40), that is:

$$A^{tax} = A_i^{tax} + A_r^{tax} = \frac{\hat{\delta}\left(r+\delta_k\right)}{r\left(\hat{\delta}+r+\pi\right)}. \tag{A.41}$$

As mentioned, the declining balance method is typically applied to investment in machinery, so from (A.37), (A.38) and (A.41) we get:

Present value of accelerated depreciation in case of investment in machinery:

$$a = A^{tax} - A^{true} = \frac{r\left(\hat{\delta}-\delta_k\right)-\pi\delta_k}{r\left(\hat{\delta}+r+\pi\right)}. \tag{A.42}$$

In the absence of inflation ($\pi = 0$) the present value of accelerated depreciation will obviously be positive if the depreciation rate for tax purposes ($\hat{\delta}$) exceeds the true real rate of economic depreciation (δ_k), as shown by (A.42). However, since depreciation for tax purposes is calculated on a historical cost basis, inflation will erode the real value of depreciation allowances over time, so if the inflation rate is sufficiently high, we see from (A.42) that our parameter may take a negative value even if $\hat{\delta} > \delta_k$.

Under the straight-line method of depreciation, the annual depreciation allowance is calculated as a fraction $1/n$ of the original acquisition cost of the asset, where n is the number of years over which the asset is allowed to be written down. For example, if $n = 25$ the annual depreciation allowance is 4 percent of the acquisition cost, implying that the asset will have been written down to zero after 25 years. At the time of acquisition, the present value of the depreciation allowances generated by the initial investment is therefore equal to:

$$A_i^{tax} = \int_0^n \left(\frac{1}{n}\right) e^{-(r+\pi)t}\, dt = \frac{1-e^{-(r+\pi)n}}{n\left(r+\pi\right)}. \tag{A.43}$$

At time zero, the present value of all future depreciation for tax purposes on the reinvestment undertaken at time t to maintain the real value of the asset is

$$\delta_k e^{\pi t} \cdot e^{-(r+\pi)t} \cdot A_i^{tax} = \delta_k A_i^{tax} e^{-rt},$$

where A_i^{tax} is given by (A.43). It follows that the present value of depreciation on all reinvestment undertaken after the time of the initial investment is

$$A_r^{tax} = \int_0^{\infty} \delta_k A_i^{tax} e^{-rt} dt = \frac{\delta_k A_i^{tax}}{r}, \qquad \text{(A.44)}$$

so by adding (A.43) and (A.44) we optain the present value of all the depreciation allowances associated with the investment:

$$A^{tax} = A_i^{tax} + A_r^{tax} = \left(\frac{r+\delta_k}{r} \right) \left[\frac{1-e^{-(r+\pi)n}}{n(r+\pi)} \right]. \qquad \text{(A.45)}$$

Recalling that the straight-line method is typically applied to buildings, we can now use (A.37), (A.38) and (A.45) to derive

The present value of accelerated depreciation in case of investment in buildings:

$$a = A^{tax} - A^{true} = \frac{1}{r} \left[\left(\frac{r+\delta_k}{r+\pi} \right) \left(\frac{1-e^{-(r+\pi)n}}{n} \right) - \delta_k \right]. \qquad \text{(A.46)}$$

The cost of capital under an Allowance for Corporate Equity

To eliminate tax distortions to investment, Sørensen (2010) proposes to introduce a so-called Allowance for Corporate Equity (ACE). Under such a corporate tax system, companies are allowed to deduct an imputed nominal market interest rate $r+\pi$ on the net equity recorded in their tax accounts. This section shows that the ACE system will be neutral towards the investment decisions of widely held firms, assuming that the imputed return on corporate equity equals the rate at which the suppliers of finance to the company discount its future cash flows.

In case of a 1 krona *equity-financed investment in machinery* undertaken at time zero, the total capital allowance granted to the firm in period *t* is

$$\overbrace{\hat{\delta}e^{-\hat{\delta}t}}^{\substack{\text{depreciation}\\ \text{allowance}}}+\overbrace{(r+\pi)e^{-\hat{\delta}t}}^{\substack{\text{ACE}\\ \text{allowance}}},$$

since the ACE allowance is calculated on the basis of the value $e^{-\hat{\delta}t}$ to which the asset is written down in the tax accounts. Hence the present value of the total allowances generated by the initial investment is

$$A_i^{tax}=\int_0^{\infty}\left(\hat{\delta}+r+\pi\right)e^{-\left(\hat{\delta}+r+\pi\right)t}dt=\frac{\hat{\delta}+r+\pi}{\hat{\delta}+r+\pi}=1. \qquad \text{(A.47)}$$

In year t the company reinvests the nominal amount $\delta_k e^{\pi t}$ in order to maintain the real value of the asset, so the present value (at time zero) of the allowances generated by the reinvestment undertaken at time *t* is

$$\left(\hat{\delta}+r+\pi\right)\int_t^{\infty}\delta_k e^{\pi t}\cdot e^{-\hat{\delta}(u-t)}\cdot e^{-(r+\pi)u}du=\left(\hat{\delta}+r+\pi\right)\delta_k e^{\left(\pi+\hat{\delta}\right)t}\int_t^{\infty}e^{-\left(\hat{\delta}+r+\pi\right)u}du=\delta_k e^{-rt}.$$

Thus the present value of the total allowances generated by all reinvestment undertaken from time zero and onwards is:

$$A_r^{tax}=\int_0^{\infty}\delta_k e^{-rt}dt=\frac{\delta_k}{r}. \qquad \text{(A.48)}$$

It follows that the total allowances triggered by the initial investment and the subsequent reinvestments have a present value of

$$A_i^{tax}+A_r^{tax}=1+\frac{\delta_k}{r}, \qquad \text{(A.49)}$$

so from (A.38) and (A.49) we get

$$a = A^{tax} - A^{true} = 1. \tag{A.50}$$

Inserting (A.50) into (A.5), we obtain the cost of capital for equity-financed investment in machinery by a widely held corporation:

$$\rho_w^e - \delta_k = r. \tag{A.51}$$

We see that the cost of capital equals the international real interest rate, as would be the case in the absence of taxes.

If the investment in machinery is instead financed by *debt*, the initial borrowing of 1 krona will ceteris paribus reduce the company's recorded equity base by 1 krona in all future periods, resulting in a drop in the ACE allowance amounting to $r+\pi$ in each future period. Instead of being given by (A.47), the present value of the capital allowances generated by the initial investment and the associated initial borrowing therefore becomes

$$A_i^{tax} = \int_0^{\infty} \left(\hat{\delta} + r + \pi\right) e^{-\left(\hat{\delta}+r+\pi\right)t} dt - \int_0^{\infty} \left(r+\pi\right) e^{-(r+\pi)t} dt = 1 - 1 = 0. \tag{A.52}$$

In each future period t, the firm increases its nominal debt by the amount $\pi e^{\pi t}$ as it borrows against the increase in the nominal value of its asset to keep the debt-asset ratio constant. The reduction in the present value of future ACE allowances caused by the additional debt incurred in period t is

$$\left(r+\pi\right) \int_t^{\infty} \pi e^{\pi t} \cdot e^{-(r+\pi)u} du = \pi e^{-rt},$$

so the present value of the reduction in ACE allowances caused by all the borrowing undertaken after the time of the initial investment is:

$$A_d^{tax} = \int_0^{\infty} \pi e^{-rt} dt = \frac{\pi}{r}. \tag{A.53}$$

The present value of the allowances generated by the reinvestment undertaken after time zero is still given by (A.48). From (A.38), (A.48), (A.52) and (A.53) we then find

$$a = A_i^{tax} + A_r^{tax} - A_d^{tax} - A^{true} = 0 + \frac{\delta_k}{r} - \frac{\pi}{r} - \frac{\delta_k}{r} = -\frac{\pi}{r}, \quad \text{(A.54)}$$

which may be inserted into (A.7) to give the cost of capital for a debt-financed investment in machinery undertaken by a widely held corporation:

$$\rho_w^d - \delta_k = r. \quad \text{(A.55)}$$

Again we see that the ACE system ensures that taxation is neutral towards investment, since no parameters of the tax system appear in (A.55).

Consider next a 1 krona *equity-financed investment in a building* which is written off over a period of n years in accordance with the straight-line method. In period $t \leq n$, the total capital allowance generated by the initial investment undertaken at time zero will then be

$$\overbrace{\frac{1}{n}}^{\text{Depreciation allowance}} + \overbrace{(r+\pi)\left(\frac{n-t}{n}\right)}^{\text{ACE allowance}},$$

so the present value of the allowances stemming from the initial investment is

$$A_i^{tax} = \int_0^n \left[\frac{1}{n} + (r+\pi)\left(\frac{n-t}{n}\right)\right] e^{-(r+\pi)t} dt = \int_0^n \left(\frac{1}{n} + r + \pi\right) e^{-(r+\pi)t} dt - \left(\frac{r+\pi}{n}\right) \int_0^n t \cdot e^{-(r+\pi)t} dt \quad \text{(A.56)}$$

$$= \left(\frac{1}{r+\pi}\right)\left(\frac{1}{n} + r + \pi\right)\left[1 - e^{-(r+\pi)n}\right] - \left(\frac{r+\pi}{n}\right) \int_0^n t \cdot e^{-(r+\pi)t} dt.$$

Using integration by parts, one finds that

$$\int_0^n t \cdot e^{-(r+\pi)t} dt = \left(\frac{1}{r+\pi}\right)\left[\frac{1 - e^{-(r+\pi)n}}{r+\pi} - n \cdot e^{-(r+\pi)n}\right],$$

which may be inserted into (A.56) to give

$$A_i^{tax} = 1. \tag{A.56}$$

From (A.56) it follows that the present value of all allowances generated by the reinvestment undertaken to maintain the real value of the building after time zero is

$$A_r^{tax} = \int_0^{\infty} \delta_k e^{\pi t} \cdot A_i^{tax} \cdot e^{-(r+\pi)t} dt = \frac{\delta_k A_i^{tax}}{r} = \frac{\delta_k}{r}. \tag{A.57}$$

Using (A.38), (A.56) and (A.57), we find the present value of capital allowances for tax purposes in excess of true economic depreciation to be

$$a = A_i^{tax} + A_r^{tax} - A^{true} = 1 + \frac{\delta_k}{r} - \frac{\delta_k}{r} = 1. \tag{A.58}$$

Substituting (A.58) into (A.5), the reader may verify that the cost of capital for widely held corporations undertaking equity-financed investments in buildings will once again be equal to the international real interest rate, $\rho_w^e - \delta_k = r$.

In the alternative case where the investment in buildings is *debt-financed*, the present value of the capital allowances generated by the initial investment is reduced by the amount $\int_0^{\infty} (r+\pi) e^{-(r+\pi)t} dt = 1$ because of the loss of ACE-allowances associated with the use of debt rather than equity finance. From this observation plus (A.56) it follows that $A_i^{tax} = 0$. The present values of the allowances flowing from the reinvestment and the additional borrowing undertaken after the time of the initial investment are still given by (A.57) and (A.53), respectively. It follows that *a* is still given by (A.54), and when this expression for *a* is inserted in (A.7), one finds once more that $\rho_w^d - \delta_k = r$.

The above analysis shows that, regardless of the type of asset and the mode of finance, the ACE system will ensure that the tax system is neutral towards the investment decisions of widely held corporations.

Effective personal tax rates on labour income and capital gains

The formula for the cost of capital for investment financed by the retained earnings of closely held corporations includes the effective personal tax rate on accrued capital gains on shares (τ^g). This rate is lower than the statutory tax rate on realized gains (τ^{sg}), since taxpayers can defer their capital gains tax until the time of realization. Specifically, if a nominal capital gain of one unit accrues to the shareholder at time zero, and if he realizes a fraction γ of his remaining gain in all subsequent periods, the effective tax rate on the accrued gain – defined as the present value of the future tax paid on realizations – may be found as

$$\tau^g = \int_0^\infty \tau^{sg}\gamma \cdot e^{-\left(\gamma + r^d + \pi\right)t} dt = \frac{\tau^{sg}\gamma}{\gamma + r^d + \pi}. \tag{A.59}$$

The parameter γ may alternatively be interpreted as the fraction of shareholders who realize (all of) their accrued gains in any given year. In that case the average holding period for shares is given by $1/\gamma$. For example, if $\gamma = 0.2$, the average investor holds his shares for five years before selling them. If the investor's nominal after-tax discount rate ($r^d + \pi$) is 0.1, it then follows from (A.59) that the effective tax rate on accrued capital gains is only two thirds of the statutory tax rate on realized gains.

When applying formula (A.59) to the case of a qualified shareholder, one must account for the fact that any capital gain exceeding the normal dividend is taxed at the personal labour income tax rate τ^{pw} rather than at the reduced capital income tax rate for qualified shareholders. For this category of shareholders, we therefore calculate the statutory tax rate on realized capital gains as

$$\tau^{sg} = f \cdot \tau^c + \left(1 - f\right) \cdot \tau^{pw} \tag{A.60}$$

where f is the estimated fraction of the gain which is taxed as capital income.

The effective total tax rate on labour income appearing in the formulas for the cost of capital for sole proprietorships and closely held companies includes the social security tax as well as the personal labour income tax. If w is the marginal taxable personal labour income after deduction for social security tax, s is the tax-exclusive marginal social security tax rate, and τ_m^{pw} is the marginal personal tax rate on labour income, the total marginal effective tax rate on labour income (τ_m^w) is found as

$$\tau_m^w = \frac{w(1+s) - w(1-\tau_m^{pw})}{w(1+s)} = \frac{s + \tau_m^{pw}}{1+s} \qquad \text{(A.61)}$$

Appendix B
The interest elasticities of labour supply and saving in the life cycle model

When applying formula (4.28) to calculate the marginal deadweight loss from a rise in the tax rate on savings income, we need information on the compensated interest elasticities of labour supply and savings about which little is known. This appendix shows how these elasticities are linked to other variables on which more information may be available.

The compensated interest elasticity of labour supply

Using 'tilde' superscripts to denote compensated variables, the symmetry properties of the Slutsky matrix imply that

$$-\frac{\partial \tilde{L}}{\partial p} = \frac{\partial \tilde{C}_2}{\partial w} \qquad \Leftrightarrow \qquad \frac{\partial \tilde{L}}{\partial p}\frac{p}{L} = -\frac{\partial \tilde{C}_2}{\partial w}\frac{p}{L}. \qquad \text{(B.1)}$$

The consumer's second-period budget constraint in our two-period life cycle model is

$$C_2 = \left[1 + r\left(1 - t^r\right)\right]S + B_2 / P \qquad \Leftrightarrow \qquad S = p\left(C_2 - B_2 / P\right) \quad \text{(B.2)}$$

from which it follows that

$$\frac{\partial \tilde{S}}{\partial w} = p\frac{\partial \tilde{C}_2}{\partial w}, \qquad \text{(B.3)}$$

since Section 4 of the main text assumed that consumers are compensated for a change in w via an adjustment of B_1 rather than B_2. Equations (B.1) and (B.3) imply

$$\frac{\partial \tilde{L}}{\partial p}\frac{p}{L} = -\frac{\partial \tilde{S}}{\partial w}\frac{1}{L} = -\varepsilon_w^S \frac{S}{wL}, \qquad \varepsilon_w^S \equiv \frac{\partial \tilde{S}}{\partial w}\frac{w}{S}, \tag{B.4}$$

where ε_w^S is the compensated elasticity of saving with respect to the marginal after-tax real wage. From (B.4) plus (4.11) and (4.13) in Section 4 of the main text we get

$$\frac{\partial \tilde{L}}{\partial p}\frac{p}{L} = -\varepsilon_w^S \frac{s\left(wL + B_1 / P\right)}{wL} = -\varepsilon_w^S s\left(\frac{1-t^w+b_1}{1-t^w}\right) = -\varepsilon_w^L s, \tag{B.5}$$

where it is recalled that s is the average savings rate of working households.

The relative price of future consumption is $p \equiv 1/\left(1+r^a\right)$, so the change in p caused by a change in the after-tax real interest rate is

$$\frac{dp}{dr^a} = \frac{-1}{\left(1+r^a\right)^2}. \tag{B.6}$$

Using this relationship along with (B.5), we find the compensated net interest elasticity of labour supply (ε_r^L) to be

$$\varepsilon_r^L \equiv \frac{\partial \tilde{L}}{\partial r^a}\frac{r^a}{L} = \frac{\partial \tilde{L}}{\partial p}\frac{p}{L}\frac{r^a}{p}\frac{dp}{dr^a} = \frac{\partial \tilde{L}}{\partial p}\frac{p}{L}\cdot\left(\frac{-r^a}{1+r^a}\right) \quad \Rightarrow$$

$$\varepsilon_r^L \equiv \left(\frac{r^a}{1+r^a}\right)s\varepsilon_w^L, \tag{B.7}$$

which is identical to (4.29) in Section 4.

The compensated interest elasticity of saving

In Section 4 we assumed that the consumers are compensated for a change in the relative price of future consumption through an adjustment of the present value of pensions, pB_2. According to (4.4) in section 4 the real value of the compensation is thus given by

$$\frac{\partial(pB_2)}{\partial p}\frac{1}{P}=\frac{\partial E}{\partial p}\frac{1}{P}=C_2. \tag{B.8}$$

From (B.2), (B.6) and (B.8) it follows that

$$\frac{\partial\tilde{S}}{\partial r^a}=-\left(\frac{1}{1+r^a}\right)^2\left[C_2-\frac{B_2}{P}+p\frac{\partial\tilde{C}_2}{\partial p}-\overbrace{\frac{\partial(pB_2)}{\partial p}\frac{1}{P}}^{=C_2}\right]=\left(\frac{1}{1+r^a}\right)^2\left(\frac{B_2}{P}-p\frac{\partial\tilde{C}_2}{\partial p}\right)\quad\Rightarrow$$

$$\frac{\partial\tilde{S}}{\partial r^a}\frac{r^a}{S}=\left(\frac{r^a}{1+r^a}\right)\left(\frac{pB_2/P}{S}-\frac{pC_2}{S}\frac{\partial\tilde{C}_2}{\partial p}\frac{p}{C_2}\right). \tag{B.9}$$

From (B.2) we have $pC_2=S+pB_2/P$. Inserting this into (B.9) and rearranging, we end up with

$$\varepsilon_r^S=\left(\frac{r^a}{1+r^a}\right)\left[b+\varepsilon_p^C(1+b)\right], \tag{B.10}$$

$$\varepsilon_r^S\equiv\frac{\partial\tilde{S}}{\partial r^a}\frac{r^a}{S},\qquad \varepsilon_p^C\equiv\frac{\partial\tilde{C}_2}{\partial p}\frac{p}{C_2},\qquad b\equiv\frac{B_2/P}{(1+r^a)S},$$

where ε_r^S is the compensated elasticity of saving with respect to the after-tax real interest rate, ε_p^C is the compensated own-price elasticity of demand for future consumption, and b is the amount of old-age consumption financed by public pensions relative to the amount that is financed by previous savings.

When the consumer is *not* compensated for a change in the after-tax real interest rate, it follows from (B.2) and (B.6) that the effect on saving will be

$$\frac{\partial S}{\partial r^a} = -\left(\frac{1}{1+r^a}\right)^2 \left(C_2 - \frac{B_2}{P} + p\frac{\partial C_2}{\partial p}\right) \Rightarrow \frac{\partial S}{\partial r^a}\frac{r^a}{S} = \left(\frac{r^a}{1+r^a}\right)\left(\frac{p\left(B_2/P - C_2\right)}{S} - \frac{pC_2}{S}\frac{\partial C_2}{\partial p}\frac{p}{C_2}\right) \Rightarrow$$

$$\hat{\varepsilon}_r^S = \left(\frac{r^a}{1+r^a}\right)\left[\hat{\varepsilon}_p^C\left(1+b\right)-1\right], \qquad \hat{\varepsilon}_r^S \equiv \frac{\partial S}{\partial r^a}\frac{r^a}{S}, \qquad \hat{\varepsilon}_p^C \equiv \frac{\partial C_2}{\partial p}\frac{p}{C_2}, \quad \text{(B.11)}$$

where $\hat{\varepsilon}_r^S$ is the uncompensated net interest elasticity of saving, and $\hat{\varepsilon}_p^C$ is the uncompensated own-price elasticity of demand for future consumption.

We now wish to identify the link between ε_r^S and $\hat{\varepsilon}_r^S$. For this purpose we start by rewriting the consumer's lifetime budget constraint (1.24) as

$$C_1 + pC_2 + w\left(1-L\right) = Y, \qquad Y \equiv \frac{W\left(1-t^w\right)+B_1+pB_2}{P}. \quad \text{(B.12)}$$

In (B.12) we have normalized the consumer's total time endowment to unity, so $w\left(1-L\right)$ is the consumption of leisure, and Y is the present value of potential lifetime income, that is, the income which could be earned if the consumer worked during all of the first period of life. The standard Slutsky decomposition implies that

$$\frac{\partial C_2}{\partial p} = \frac{\partial \tilde{C}_2}{\partial p} - C_2\frac{\partial C_2}{Y} \quad \Rightarrow$$

$$\varepsilon_p^C = \hat{\varepsilon}_p^C - c\varepsilon_Y^C, \qquad \hat{\varepsilon}_Y^C \equiv \frac{\partial C_2}{\partial Y}\frac{Y}{C_2}, \qquad c \equiv \frac{pC_2}{Y}, \quad \text{(B.13)}$$

where $\hat{\varepsilon}_Y^C$ is the income elasticity of demand for future consumption, and c is the fraction of potential lifetime income spent on consumption during retirement. From (B.11) and (B.13) it follows that

$$\varepsilon_p^C = \frac{\left(\frac{1+r^a}{r^a}\right)\hat{\varepsilon}_r^S + 1}{1+b} - c\varepsilon_Y^C. \quad \text{(B.14)}$$

Inserting (B.14) into (B.10), we obtain

$$\varepsilon_r^S = \hat{\varepsilon}_r^S + \left(\frac{r^a}{1+r^a}\right)\left(1+b\right)\left(1-c\varepsilon_I^C\right), \quad \text{(B.15)}$$

which is seen to be identical to equation (4.30) in Section 4. To obtain the expression for b stated below (4.30), we used (4.11) to eliminate S from the definition of b given in (B.10) and exploited the definitions of the variables b_1 and b_2.

Appendix C
Calibration to Swedish data

This appendix illustrates how the various parameters may be quantified with the purpose of applying the formulas derived in this paper to estimate the efficiency losses from taxation in Sweden.

Income data

The data reported in this section were taken (or estimated) from the Swedish national income accounts produced by Statistiska centralbyrån, from "Beräkningskonventioner 2010", Finansdepartementet, and from the FRIDA database. Unless otherwise indicated, all figures refer to 2008.

$WL =$ aggregate wage bill for the total economy, including employers' social security contributions: 1863 bill. SEK (Aggregate wage bill for the private sector: 1323 bill. SEK)

$(\rho - \delta_k)K =$ aggregate net business profits earned on domestic business investment (gross profit minus economic depreciation): 258 bill. SEK

$rPS =$ aggregate income from wealth earned by domestic residents (including imputed returns to owner-occupied housing and returns to savings channelled through pension funds and life insurance companies etc.): 317 bill. SEK

$$b_1 \equiv \frac{B_1}{WL} = \frac{\text{public after-tax transfers to people of working age}}{\text{aggregate wage bill}}$$

$$+\overbrace{\text{marginal labour income tax rate}}^{t^w} \text{ - average labour income tax rate} = \frac{155}{1863} + 0.476 - 0.332 = 0.227.$$

$$b_2 \equiv \frac{B_2}{WL} = \frac{\text{public after-tax transfers to people above working age}}{\text{aggregate wage bill}} = \frac{160}{1863} = 0.086.$$

$\rho K =$ total business profits before depreciation and interest: 543 bill. SEK

$\rho_c K_c =$ profits before depreciation and interest in closely held firms (unincorporated firms plus closely held corporations): 148 bill. SEK

$\rho_{cc} K_{cc} =$ profits before interest and depreciation in closely held corporations: 127 bill. SEK

$\rho_{cp} K_{cp} =$ profits before interest and depreciation in unincorporated firms: 21 bill. SEK

$(r+\pi) PS^I =$ nominal return to financial wealth held by institutional investors (pension funds and life insurance companies): 101 bill. SEK

$(r+\pi) PS^F =$ nominal return to financial wealth held directly by households (wealth not held through institutional investors): 94 bill. SEK

Explanatory comments: Data on the variables $\rho_c K_c$, $\rho_{cc} K_{cc}$ and $\rho_{cp} K_{cp}$ are not directly available. These numbers were therefore estimated by decomposing the national income figure for total business profits (ρK) in accordance with the estimated share of each sector in total profits, based on a simple average of the figures in columns 1 and 2 in the following table:

Table A.2.1 Distribution of economic activity across alternative forms of business organization in Sweden, 2005

Type of firm	1. Percent of total turnover	2. Percent of total wage bill	3. Average of 1. and 2.
Widely held public and private corporations (noterade och onoterade aktiebolag)	75.2	70.2	72.7
Closely held corporations (fåmansföretag)	19.6	27.2	23.4
Sole proprietorships and partnerships (enskilda näringsidkare och handelsbolag)	5.2	2.6	3.9

Source: FRIDA database.

Table 4.3 on p. 82 in "Beräkningskonventioner 2010" provides an estimate of the imputed pre-tax return to institutional saving, assumed to correspond to our variable $(r+\pi)PS^I$. Our variable $(r+\pi)PS^F$ is estimated as the sum of taxable net capital income accruing directly to households (= 84 bill. SEK, according to Table 4.1 on p. 77 in "Beräkningskonventioner 2010") and the imputed return to the value of shares held in Mutual Funds (investeringsfonder och investmentföretag), estimated as 0.015x(442.6+226)=10 bill. SEK on the basis of Table 4.2 on p. 80 in "Beräkningskonventioner 2010". Note that the low imputed rate of return on Mutual Funds is only meant to capture expected capital gains, since dividends paid out from the funds are taxed as capital income in the hands of household investors. Assuming an average real pre-tax rate of return of 5 percent and an inflation rate of 2 percent, the implied estimate of the total *real* return to financial savings is (5/(5+2))x(101+94) = 139 bill. SEK. In addition, households earn a real return on their housing wealth in the form of the value of the housing services generated by the stock of owner-occupied housing. In 2008, the imputed rentals of owner-occupiers (nyttjandevärde av småhus och fritidshus) amounted to 178 bill. SEK. Assuming that the average nominal capital gain on owner-occupied housing equals the average rate of inflation, these imputed rentals also measure the average real rate of return on this form of saving. Hence the total estimated real rate of return to saving is $rPS=139+178=317$ bill. SEK.

Finally, the marginal and average effective tax rates on labour income used in the estimation of b_1 and b_2 include the estimated tax component of social security contributions. Means-tested transfers are counted as negative taxes, and the effect of the phasing-out of these transfers as income goes up is included in the measure of the effective marginal tax rate. The tax rates are weighted averages across the whole working population, estimated by the Swedish Ministry of Finance.

Data on consumption

The data in this section are taken from the Swedish national income accounts and from revenue figures provided by the Swedish Ministry of Finance. Consumption expenditures include indirect taxes, and all figures refer to 2008.

$PC =$ total private consumption: 1467 bill. SEK

$P_H C_H =$ total consumption of housing services: 293 bill. SEK

$p_H H =$ consumption of owner-occupied housing services: 178 bill. SEK

$p_h h =$ consumption of rental housing services: 115 bill. SEK

$p_1 x_1 =$ consumption of non-housing goods subject to 25% VAT: 742.2 bill. SEK

$p_2 x_2 =$ consumption of non-housing goods subject to 12% VAT: 193.5 bill. SEK

$p_3 x_3 =$ consumption of non-housing goods subject to 6% VAT: 58.5 bill. SEK

$p_4 x_4 =$ consumption of non-housing goods subject to 0% VAT: 29.0 bill. SEK

$p_5 x_5 =$ consumption of VAT-exempt non-housing goods subject to an implicit 17.6% VAT rate (due to the non-deductibility of input VAT): 57.5 bill. SEK

$p_6 x_6 =$ consumption of VAT-exempt non-housing goods subject to an implicit 14.9% VAT rate (due to the non-deductibility of input VAT): 79.3 bill. SEK

Tax rates

The data and estimates summarized in this section are based on information provided by the Swedish Ministry of Finance.

VAT rates: 0.25, 0.12, 0.06, 0.0, 0.176 (implicit rate), 0.149 (implicit rate).

t_e^1 = average excise tax rate on goods subject to the standard 25% VAT rate: 0.25

t^H = VAT rate on sale of newly constructed housing units: 0.25

τ^H = effective property tax rate on owner-occupied housing, measued relative to the current market value of the property: 0.0068

τ^h = effective property tax rate/subsidy rate on rental housing, measured relative to the current market value of the property: 0.0025

t^w = average value of the marginal direct tax rate on gross labour income, including social security taxes paid by employers and employees: 0.476

t_s^I = statutory tax rate on nominal return to savings channelled through institutional investors (pension funds and life insurance companies): 0.15

t_s^F = statutory marginal personal tax rate on nominal capital income: 0.30

Explanatory comments: The first four VAT rates stated above are the statutory rates specified by the VAT law for various consumption categories, including those that are zero-rated. The estimated implicit VAT rate of 17.6 percent applies to out-patient services and hospital services, certain cultural services and social protection (samhällsservice). The sale of these services is exempt from VAT but is subject to an implicit VAT bill since the VAT paid on inputs into the production of these services is non-deductible. Similarly, the VAT of 14.9 percent is an implicit rate for VAT-exempt insurance, financial services and gambling. These implicit VAT rates were estimated by the Ministry of Finance and reflect assumptions on the share of value-added in the total sales of the relevant goods and services.

The effective property tax rates τ^H and τ^h are calculated by dividing the net revenue from the various property taxes by an estimate of the current market value of owner-occupied and rental housing property, respectively. The data used in the calculations are as follows:

Taxes on owner-occupied housing, 2008 (bill. SEK)	
Property tax (fastighetsskatt, fysiska personer)	10.5
Tax on realized capital gains	13.5
Tax on deferred capital gains (ränta på uppskov)	1.1
Stamp duties (stämpelskatter, fysiska personer)	4.2
Total taxes on owner-occupied housing	29.3

Net taxes on rental housing	
Property tax	2.9
Stamp duties relating to rental property	1.1
nterest subsidies (räntebidrag)	0.8
Investment subsidies	0.4
Total net taxes on rental housing	2.8

Taxable property values, 2009 (taxeringsvärde, bill. SEK)	
Owner-occupied dwellings, excluding condominiums and farm houses (småhus)	2688
Condominiums (bostadsrätter)	377
Farm houses	159
Total taxable value of owner-occupied housing property	3223
Rental housing property	839

The capital income tax on realized capital gains on owner-occupied housing property may be deferred in so far as the gains are reinvested in a new home, but taxpayers must pay capital income tax on an imputed interest rate on their deferred gains. This is recorded as a tax on deferred gains (ränta på uppskov) in the table above.

The estimated property values summarized above are taken from the general assessment (den allmänna fastighetstaxering) undertaken by the Swedish authorities in 2009. According to the law, the assessment should reflect 75% of the estimated market value, based on the average property price level prevailing two years earlier. In principle, the market value of property in 2007 may thus be estimated by dividing the assessed figures (taxeringsvärde) by 0.75. The resulting numbers are used as a proxy for the market values in 2008. An alternative would be to calculate the effective property tax rates on the basis of revenue figures for 2007, but this procedure would fail to account for the significant drop in the property tax which took effect in 2008.

Depreciation rates

A detailed estimation of exponential economic rates of depreciation for various assets is provided by Hulten (2008). That study provided the basis for the following estimates:

δ = real annual rate of economic depreciation of housing capital: 0.02
δ_k = average real annual rate of economic depreciation of business capital: 0.09

The estimated rate of depreciation of residential capital is slightly higher than Hulten's estimate of the depreciation rate for new structures and slightly below his estimate the depreciation of additions and alterations to existing structures.

Economic depreciation rates for business assets vary considerably across asset types, making it very difficult to estimate an average depreciation rate across all assets. However, according to Hulten (op.cit.) the depreciation rates for many important types of machinery are close to 15 percent, business structures and buildings typically depreciate at a rate close to 3 percent. The assumed value of δ_k is the simple average of these two figures.

Elasticities

In the benchmark calculations in Sørensen (2010), the following assumptions on elasticities are used:

Elasticity of substitution between housing services and other goods (σ): 1.0
Elasticity of substitution between the various 'other goods' (σ_o): 1.0
Elasticity of substitution between owner-occupied housing and rental housing (σ_h): 1.5

Elasticity of substitution between institutional saving and 'free' financial saving (ϕ): 1.0
Elasticity of substitution between capital invested in widely and closely held firms (σ_k): 1.5
Elasticity of substitution between capital invested in closely held corporations and proprietorships (σ_c): 2.0
Elasticity of aggregate capital demand w.r.t. the user cost of capital (ε_ρ^K): 1.0
Compensated elasticity of labour supply w.r.t. the real marginal after-tax wage rate (ε_w^L): 0.2
Compensated elasticity of labour supply w.r.t. the real after-tax interest rate (ε_r^L): 0.023
Compensated elasticity of aggregate saving w.r.t. the real marginal after-tax wage rate (ε_w^S): 0.14
Compensated elasticity of aggregate saving w.r.t. the real after-tax interest rate (ε_r^S): 0.56

There is only scant empirical evidence from Sweden regarding the size of most of the elasticities listed above. In general, a unitary elasticity of substitution in consumption corresponds to the benchmark case of a Cobb-Douglas utility function which implies constant budget shares. The assumption that $\sigma = 1$ is thus consistent with the empirical observation that the value of the consumption of housing services relative to the value of total private consumption has been relatively stable over time. A priori one would expect a greater degree of substitutability between rental and owner-occupied housing than between housing consumption and other consumption. This is the motivation for setting σ_h at a somewhat higher value than σ .

Particular uncertainty attaches to the values of the sectoral substitution elasticities σ_k and σ_c. Fullerton and Henderson (1987) assume that the elasticity of substitution between corporate and non-corporate capital lies between 0.3 and 3.0. We assume a value for our parameter σ_k slightly below the mid point of this interval. As discussed in detail by Hagen and Sørensen (1998), the similarities between the organizational forms of proprietorships and closely held corporations are usually greater than the similarities between widely and closely held firms. Hence we have chosen a somewhat higher value of σ_c than of σ_k.

In their survey of empirical studies of the effects of tax policy on investment, Hassett and Hubbard (2002) conclude that the numerical user cost elasticity of capital demand (ε_{ρ}^{K}) is probably between 0.5 and 1.0. Above we have followed Auerbach and Kotlikoff (1987) in focusing on the "neoclassical benchmark case" where $\varepsilon_{\rho}^{K}=1$, since this is consistent with the empirical observation that the aggregate gross profit share of GDP is relatively constant over the long run.

The net wage elasticity of hourly labour supply has been subject to many empirical studies in Sweden as well as in other countries. However, when estimating the deadweight loss from taxation, it is really the elasticity of the labour income tax base with respect to the marginal net-of-tax rate (the so-called elasticity of taxable income) which is crucial, since this is the parameter determining the dynamic revenue loss from a higher marginal tax rate. Recent studies of the elasticity of taxable labour income in Sweden include Hansson (2007), Holmlund and Söderström (2008), Ljunge and Ragan (2008), and Blomquist and Selin (2009). The estimated elasticities in these studies generally vary from 0.2 to 0.5 (for women, Blomquist and Selin actually estimate elasticities of 1.0-1.4). These estimates are uncompensated elasticities including income effects as well as substitution effects on the tax base. Since the income effect must be assumed to be negative, the estimates mentioned above must be seen as lower bounds on the compensated elasticity which is relevant when calculating the deadweight loss. Hence our assumption that $\varepsilon_{w}^{L}=0.2$ is rather conservative.

Given the assumption on ε_{w}^{L}, the compensated factor supply elasticities ε_{w}^{S}, ε_{r}^{L} and ε_{r}^{S} have been estimated from equations (4.13), (4.29) and (4.30). To derive ε_{w}^{S} from (4.13), we used the data on t^{w} and b_1 reported above. When applying (4.29) and (4.30) to calculate ε_{r}^{L} and ε_{r}^{S}, we calculated the fraction $r^{a}/\left(1+r^{a}\right)$ on the assumption that the length of a time period in our life cycle model is 25 years, that the real pre-tax rate of return to saving is 5 percent per annum, and that the effective marginal tax rate on the return to saving equals the estimate obtained from (1.32), given the parameter values reported above. In applying (4.29), we assumed a savings rate . The use of (4.30) requires a number of assumptions. The value of b was chosen so as to be consistent with the estimates of b_1 ,b_2 and t^{w} reported earlier, implying $b=0.247$, given the assumed savings rate of 0.2. The uncompensated net interest elasticity of saving ($\hat{\varepsilon}_{r}^{S}$) was set to zero, consistent with the approximate long-run constancy of the savings rate. To obtain a long-run equilibrium with balanced growth, the income elasticity of demand for consumption during retirement (ε_Y^C) must be one, so we have chosen $\varepsilon_Y^C = 1$. The parameter c_Y in (4.30) is the fraction of potential lifetime income spend on consumption during retirement. Recalling that a part of potential lifetime income is spent on "buying" leisure, and accounting for the discounting of future consumption, we have set $c_Y = 0.2$ as a plausible guesstimate for this parameter.

Assumptions underlying the estimates of the user cost of business capital

The user cost of capital for the various forms of business organization in Sweden were estimated by means of the formulas derived in Appendix A. Based on current Swedish tax rules, the following parameter values were assumed:

Parameter	Value
Nominal interest rate	7%
Rate of inflation	2%
Statutory corporate income tax rate	26.3%
Personal capital income tax rate	30%
Personal tax rate on dividends and realized capital gains on unlisted shares	25%
Personal tax rate on dividends and realized capital gains on qualified shares	20%
Average holding period for shares	10 years
Average value of marginal personal tax rate on labour income	54%
Tax-exclusive social security tax rate (wage earners/sole proprietors)	31.42/29%
Sole proprietors: Imputed rate of return on equity[1]	9%
Qualified shareholders: Imputed rate of return on equity[2]	13%
Qualified shareholders: Fraction of capital gain taxed as labour income	100%
Qualified shareholders: Wage-based allowance included in normal dividend	25% of wage bill
Qualified shareholders: Marginal ratio of wage bill to capital stock	0
True real exponential rate of economic depreciation of machinery	15%
True real exponential rate of economic depreciation of buildings	3%
Declining balance rate of depreciation of machinery for tax purposes	30%
Straight-line rate of depreciation of buildings for tax purposes/tax life	4%/25 years
Share of capital stock invested in machinery	50%
Share of capital stock invested in buildings	50%
Share of investment financed by new equity	10%
Share of investment financed by retained earnings	50%
Share of investment financed by debt	40%

1. Average government bond rate + 5 percentage points.

2. Average government bond rate + 9 percentage points.

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